Pattern Recognition, Vol. 26, No. 10, pp. 1543–1547, 1993
Printed in Great Britain

0031-3203/93 $6.00 + .00
Pergamon Press Ltd

RECOVERING SCENE STRUCTURES FROM SCATTERED SURFACE POINTS

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(*Received 4 September* 1992; *in revised form* 30 *March* 1993; *received for publication* 7 *April* 1993)

Abstract—Given scattered points from range or volumetric images, methods for the recovery of scene structures are described. The structures are represented by rational Gaussian surfaces. Examples demonstrating reconstruction of three-dimensional (3D) scenes from scattered depth measurements obtained from stereo disparity, recovery of 3D shapes from noisy range data, and segmentation of volumetric images for the extraction of generalized cylinders are presented.

Scene recovery   Surface reconstruction   3D image features   Parametric surfaces   Range data

1. INTRODUCTION

In many computer vision problems, only scattered points from a scene are available and it is required to reconstruct the scene. Scene recovery from scattered points can be formulated as a surface fitting problem. Surface fitting to scattered data has been an active area of research for a long time.[1,2] Most surface formulations, however, work only on data that are sparsely spaced or single-valued. Among the surfaces that can approximate dense and multi-valued data, the energy minimizing surfaces[3–5] and the rational Gaussian (RaG) surfaces[6] may be named.

In this paper, RaG surfaces are used to represent scene structures. RaG surfaces are globally defined but are locally sensitive to given data. The effect of a data point on a surface point vanishes exponentially as the distance between the two increases. The degree of locality of a RaG surface can be controlled by what is known as the smoothness parameters of the surface.

A desirable property of RaG surfaces is that given points do not have to form a regular grid and any arrangement of the points is acceptable. The given points will be used as control vertices in a RaG surface. Since the points may be noisy, approximating surfaces will be used in an attempt to smooth out the noise. In this paper, measurements, control vertices, points, and given points will all be used to mean given scattered surface points. First, the types of data that may be used by the proposed method are described. Then, formulations for RaG surfaces are given. Next, methods for determination of the nodes of a RaG surface are described, and finally, examples of surface recovery from scattered points are presented.

2. THE DATA

In many computer vision problems, regularly-spaced points are not available, and one must recover the three-dimensional (3D) structure of a scene from scattered points. Figure 1 shows the elevation of scattered points in a terrain scene obtained by matching high-gradient edges in stereo images of the terrain. To reconstruct the terrain in 3D it is necessary to fit a surface to the points.

Figure 2 shows the x, y, and z coordinates of a panoramic range image of a girl's head obtained by a special range finder (Cyberware 4020 Laser Scanner). The range finder makes a complete revolution about an object while scanning. Corresponding entries in the three images show x, y, and z coordinates of 3D points on an object. Although points are given in arrays, they do not represent regularly-spaced points from an object. For example, in these images, many points in the upper rows correspond to the same point on top of the girl's head. If repeated points are removed, a set of scattered points will be obtained.

Figure 3(a) shows a 3D cardiac magnetic resonance image, and Fig. 3(b) shows extraction of the left and right ventricles by intensity thresholding. The voxels on the boundary of each of the ventricles represent scattered surface points. These points are used to reconstruct the ventricles.

These images show noisy measurements from scenes or their parts. We will use RaG surfaces to recover the geometry of a scene or a part of it. For the sake of completeness, next, formulas for RaG surfaces are given and their properties are summarized.

3. RAG SURFACES

3.1. *Open surfaces*

An open RaG surface[6] is defined by

$$\mathbf{P}(u, v) = \sum_{i=1}^{n} \mathbf{V}_i g_i(u, v) \quad u, v \in [0, 1] \tag{1}$$

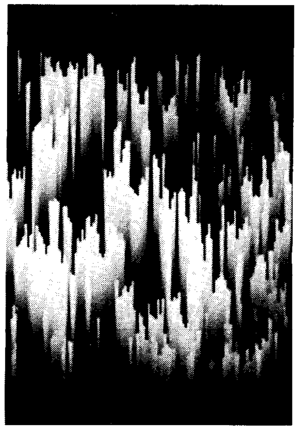
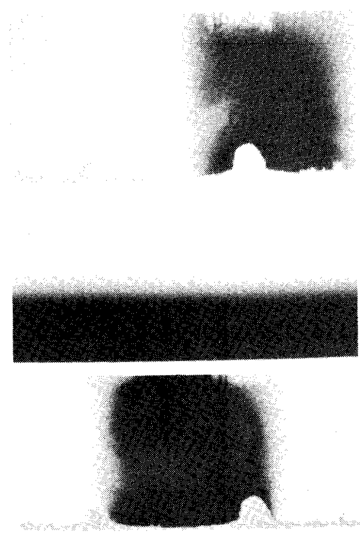


Fig. 1. Elevations of points in a terrain scene obtained by matching stereo images of the scene. Elevation values are represented by intensities with brighter points showing higher elevations. In this image, the heights of the points also show their elevations.



(a)

(b)

(c)

Fig. 2. (a)–(c) x, y, and z coordinates of points sampled from a girl's head by a special range finder (Cyberware 4020 Laser Scanner).

where

$$g_i(u,v) = \frac{W_i G_i(u,v)}{\sum_{j=1}^{n} W_j G_j(u,v)} \tag{2}$$

and

$$G_i(u,v) = \exp\{-[(u-u_i)^2 + (v-v_i)^2]/2\sigma_i^2\}. \tag{3}$$

In these formulas, n is the number of control vertices, $\mathbf{V}_i$ is the ith control vertex, W_i the weight associated with the ith control vertex, (u_i, v_i) the ith node, and $G_i(u,v)$ a 2D Gaussian with standard deviation σ_i and height one centered at (u_i, v_i). σ_i determines the smoothness of the surface in the neighborhood of the ith control vertex. If a surface is required to closely follow its control vertices, smaller standard deviations should be chosen. Since a Gaussian with a larger standard deviation has smaller high-spatial-frequency coefficients, surfaces obtained from Gaussians with larger standard deviations have smaller high-spatial-frequency coefficients. Since a surface with smaller high-spatial-frequency coefficients is smoother, we see that we can use the standard deviations of Gaussians to control the smoothness appearance of a generated surface. For that reason, the standard deviations of Gaussians are called the smoothness parameters of a surface.

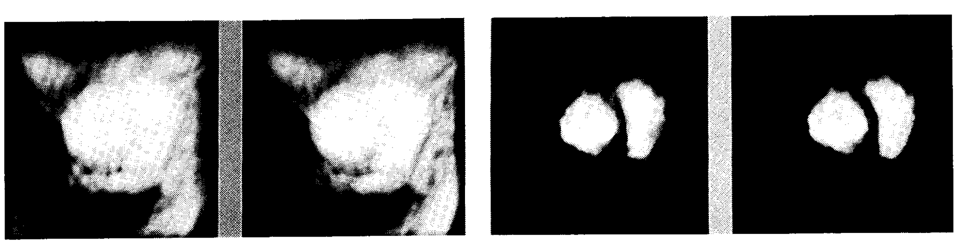
Some of the properties of RAG surfaces are: (1) they are infinitely differentiable everywhere; (2) they fall inside the convex hull of their control vertices; (3) their standard deviations can be varied to produce surfaces with different smoothness appearances; and (4) the degree of importance of the points can be incorporated into the weights, making an obtained surface pass closer to more important points.

3.2. *Generalized cylinders*

Generalized cylinders are defined as surfaces that are smoothly closed from one side. If $G_i(u,v)$ in relation (3) is replaced by

$$G_i(u,v) = \sum_{j=-\infty}^{\infty} \exp\{-\{[u-(u_i+j)]^2 + (v-v_i)^2\}/2\sigma_i^2\} \tag{4}$$

the surface obtained from relations (1) and (2) will be



(a) (b)

Fig. 3. (a) A volumetric cardiac MR image. (b) Left and right ventricles extracted by segmenting the volumetric image through intensity thresholding. These images are shown in stereo.

a generalized cylinder. The infinity in relation (4) comes from the fact that a Gaussian extends from $-\infty$ to ∞ and when the surface is closed from one side, it makes infinite cycles around it. The effect of a Gaussian, however, vanishes exponentially, and after a few cycles it may not be measurable by the accuracy of a computer.

Assuming the accuracy of a computer is ε, in formula (4) when $\exp\{-\{[u-(u_i+J)]^2+(v-v_i)^2\}/2\sigma_i^2\}<\varepsilon$ the effect of a Gaussian cannot be measured, and that happens when $J>\sigma_i\sqrt{(-2\ln\varepsilon)}$. Therefore, only when

$$J \leq \lfloor \sigma_i\sqrt{(-2\ln\varepsilon)} \rfloor \quad (5)$$

the effect of a Gaussian on a surface point is measurable. For instance, when $\varepsilon = 10^{-8}$ and $\sigma_i = 0.1$, we find $J \leq 1$. With conceivable values of ε and σ_i, we find that J is usually less than 5. Therefore, in practice, the infinity in relation (4) should be replaced by J and computed from relation (5).

4. ESTIMATION OF THE PARAMETERS OF A SURFACE

A RaG surface is defined by a set of control vertices $\{\mathbf{V}_i : i=1,\ldots,n\}$ with corresponding weights $\{W_i : i=1,\ldots,n\}$, smoothness parameters $\{\sigma_i : i=1,\ldots,n\}$, and nodes $\{(u_i, v_i) : i=1,\ldots,n\}$. To treat all the control vertices equally, all the weights of the surface should be set to the same value. Otherwise, the weights should be set proportional to the degrees of importance of the points.

The smoothness of a generated surface in the neighborhood of the ith control vertex is determined by σ_i. If measurements in a neighborhood are known to be noisy, the smoothness parameters in that neighbourhood should be increased to smooth out noise by a larger amount. If given points are not noisy, very small smoothness parameters should be used to allow the surface to closely follow the points. If it is desired to treat all areas of a surface similarly, all smoothness parameters should be set to the same parameter so that the smoothness of the entire surface could be controlled by varying a single parameter. If an application allows user interaction, this parameter may be interactively varied until a surface with a desired smoothness appearance is obtained.

Given scattered points from a surface, in addition to the weights and the smoothness parameters, the nodes of the surface should be computed also. The nodes of a surface determine the adjacency relation between the points. For some point sets, nodes can be determined easily, while for others, determination of the nodes from given points is a very difficult task.[7]

The image of Fig. 1 shows single-valued data and we can determine the nodes of the surface from the x and y coordinates of the points. Assuming the image is of size $m \times n$ with rows numbered from 0 to $(m-1)$ and columns numbered from 0 to $(n-1)$, the node assigned to the ith point is computed from

$$u_i = \frac{x_i}{m-1}, \quad v_i = \frac{y_i}{n-1} \quad (6)$$

where x_i and y_i are the row and column numbers of the ith point in the image. In this formulation, the order of the points is not important but their positions are. The parameters are determined from the positions of the points irrespective of their orders. When the nodes of an open surface are computed in this manner and parameters u and v are varied from zero to one, we will be able to traverse the surface from top to bottom and left to right.

Figure 2 shows three images representing the x, y, and z coordinates of points in 3D. Assuming j denotes the rows, varying from 0 to $(m-1)$; k denotes unique points in row j, varying from 0 to (n_j-1); and the kth point in the jth row represents the ith measurement; then (u_i, v_i) is estimated from

$$u_i = \frac{k}{n_j}, \quad v_i = \frac{j}{(m-1)}. \quad (7)$$

This formulation ensures that for $u_i = 0$ and 1, the same surface point is obtained, thus closing a surface from one side.

Figure 3(a) shows transformation of a sequence of cardiac magnetic resonance slices into an isotropic volume representation,[8] and Fig. 3(b) shows segmentation of the image by intensity thresholding. Traveling through this volumetric data along the slices, two closed contours are obtained that correspond to the left and right ventricular boundaries. Assuming j denotes the slice number varying from 0 to $(m-1)$, and k denotes the pixel number on a boundary contour in slice j, varying from 0 to (n_j-1), the node at the kth point in the jth slice can be estimated from relation (7).

Since no information about the underlying scene structures are known, it is not possible to tell how good the estimated nodes are in reconstructing a scene. If some information about a scene is known, it may be used in an iterative process to optimize the nodes.[9] In the next section, RaG surfaces with nodes as estimated above will be used to reconstruct scenes from scattered points.

5. RESULTS

Using the non-zero points in Fig. 1 as the control vertices of an open RaG surface, setting the standard deviations of all Gaussians to σ, using equal weights, and computing the nodes from relation (6), we obtain the surfaces shown in Fig. 4. Figures 4(a) and (b) show surfaces obtained by setting σ to 0.04 and 0.06, respectively. By increasing parameter σ, more scene details disappear in the constructed surface.

Fitting a generalized cylinder to the data of Fig. 2 with equal weights, standard deviations of all Gaussians equal to 0.03, and nodes as computed from relation (7), we obtain the surface shown in Fig. 5. Different views of this surface are depicted. The surface that represents the girl's head is open from the top and bottom, but is closed from the sides.

Two generalized cylinders were fitted to voxels on the boundaries of the left and right ventricles shown

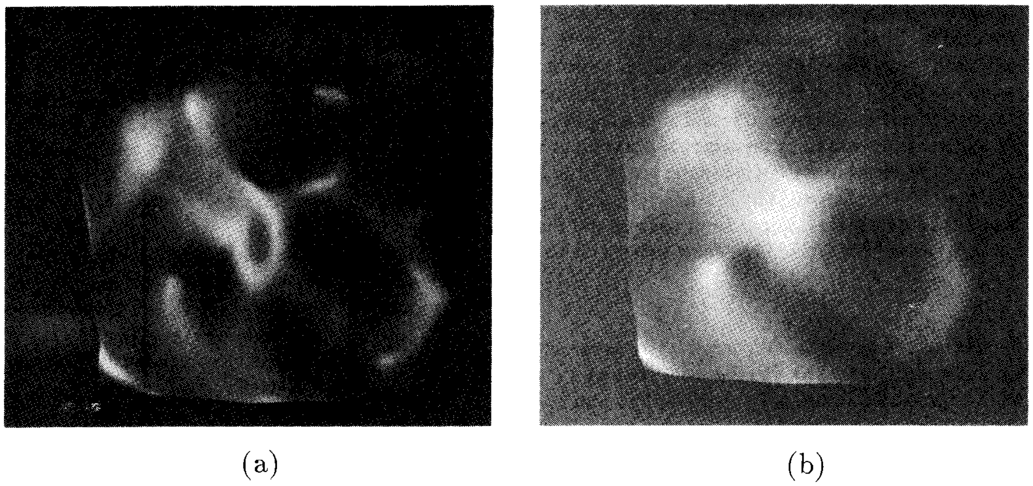


Fig. 4. (a), (b) Reconstruction of the terrain scene by fitting an open RaG surface to the data of Fig. 1 with standard deviations of all Gaussians equal to 0.04 and 0.06, respectively. All weights were set to the same value and the nodes of the surface were computed from relation (6).

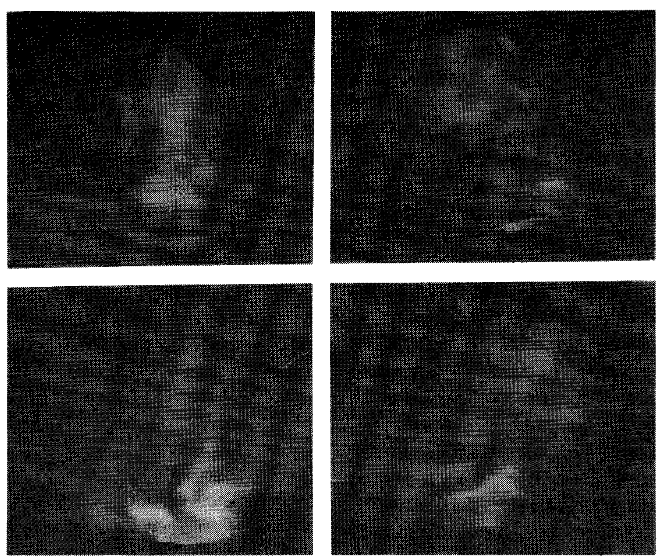


Fig. 5. Reconstruction of the girl's head by fitting a generalized cylinder to the data of Fig. 2 with equal weights and nodes as given by relation (7). The standard deviations of all Gaussians were equal to 0.03.

in Fig. 3(b). The weight at a voxel was taken to be the 3D gradient magnitude[10] at the voxel. Weights were selected in this manner because a point with a higher gradient magnitude shows a boundary point with a higher degree of confidence. Estimating the nodes of the surface from relation (7) and setting the standard deviations of all Gaussians to 0.05, the surfaces shown in Fig. 6 were obtained. These surfaces represent generalized cylinders that are smoothly closed from one side but are open from the other.

In these experiments, the standard deviations of Gaussians were interactively selected to obtain visually pleasing surfaces. If some information about the surface to be recovered is known, parameter σ may be

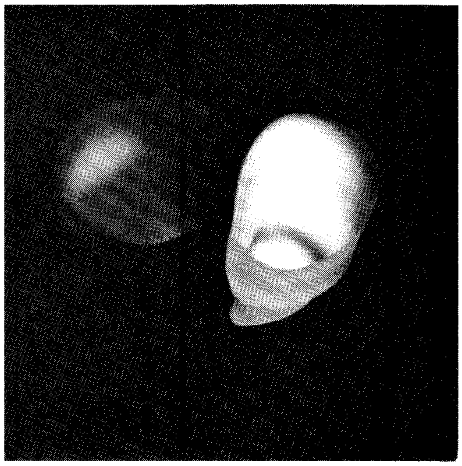


Fig. 6. Two generalized cylinders representing points on the ventricular boundaries of Fig. 3(b). The weights associated with the points were set to the 3D gradient magnitudes of the points, and the nodes of the surfaces were computed from relation (7). The standard deviations of all Gaussians were equal to 0.05.

computed automatically by iteratively changing its value until a minimum is reached in a predefined error criterion.

6. CONCLUSIONS

Recovering scene structures from scattered 3D points is required in many computer vision applications. In the proceeding sections a new method for the reconstruction of scene structures using RaG surfaces was described. Examples demonstrating recovery of terrain scenes from scattered depth measurements, and recovery of cylindrical objects in volumetric data were shown. The major challenge in the recovery of arbitrary structures from scattered 3D points would be the determination of the adjacency relation between the points or the computation of the nodes of the approximating surface.

The standard deviations of Gaussians in a RaG surface control the shape of the surface. By increasing the standard deviations of Gaussians, a smoother surface is obtained. Larger standard deviations should be used when noise in the data is significant. If given points are not noisy and the surface to be generated should follow the points closely, very small standard deviations should be used. If the standard deviations of all Gaussians in a surface are set to a common parameter, the shape of an entire surface can be controlled by a single parameter. The proper value of the common parameter can then be determined either interactively by changing its value and observing the shape of the obtained surface, or automatically by iteratively varying its value until a minimum is reached in a predefined error criterion.

Acknowledgements—This work was supported in part by the American Heart Association of Metropolitan Chicago. Images of Figs 2 and 3(a) were provided by Mr Paul Neumann of Biomedical Visualization Laboratory, University of Illinois at Chicago, and Dr David A. Turner of Magnetic Resonance Imaging, Presbyterian-St. Luke's Hospital.

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