EE321 Third Homework Assignment

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1 Laplace Transform

1. Find the **Laplace transforms** of the following functions: (either using direct integration or using convolution, time–shifting, and other properties with transform table)

   (a) $f_1(t) = (2 + e^{-2t}) \cdot u(t-2)$

   (b) $f_2(t) = (t-2)^2 \cdot \delta(t)$

   (c) $f_3(t) = 2u(t) * 3e^{-3t}u(t)$

   (d) $f_4(t) = 2[u(t-1) - u(t-4)]$

   (e) $f_5(t) = tu(t-1)$

2. **Find Laplace Transform and the region of convergence using direct integration:**

   (a) $x_1(t) = u(t) - u(t-3)$

   (b) $x_2(t) = (e^{-3t} - 3e^{-t})u(t)$

   (c) $x_3(t) = e^{-3t}\cos(20t+2)u(t)$

3. **Initial and final value theorem:** Determine the initial and final values of the signals whose Laplace transforms are given. If the final values are not defined, state why.

   (a) $F_1(s) = \frac{2}{s^3+s}$

   (b) $F_2(s) = \frac{5s}{s^2+2s+5}$

   (c) $F_3(s) = \frac{3s^2+4s+2}{s^3+2s^2+s+2}$

   (d) $F_4(s) = \frac{3}{s^2-s}$

4. Find the **inverse Laplace transforms** of the following functions:

   (a) $F_1(s) = \frac{s+3}{s(s^2+9s+14)}$

   (b) $F_2(s) = \frac{(s+3)e^{-2s}}{(s^2-2s-3)}$

   (c) $F_3(s) = \frac{s+2e^{-s}}{s^2+2s+5}$

   (d) $F_4(s) = \frac{s+1}{(s+2)^2(s^2+4s+5)}$

5. **Solving differential equations:**

(a) A LTI system is described by the following differential equation.

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2\frac{dx(t)}{dt} + x(t)$$

with the initial conditions: $y(0) = 2$, $\dot{y}(0) = -1$. i) Find transfer function $H(s)$ by observation; ii) Find the zero input response using Laplace transform; iii) Find the zero state response to $10u(t)$ using Laplace transform; iv) Find the total response of the system to the signal $10u(t)$.

(b) The transfer function of a LTI system is given as

$$H(s) = \frac{s+2}{s^2+4s+5}$$

and its output initial condition is $y(0) = 1$, $\dot{y}(0) = -2$. i) Find the zero input response using Laplace transform; ii) Find the zero state response to input signal $2e^{-2t}u(t)$ using Laplace transform; iii) What is the total response.

(c) Given a circuits where the switch is in an opened position for a long time before $t = 0$ when it is closed instantaneously. Find the total response of $y(t)$ for $t \geq 0$.

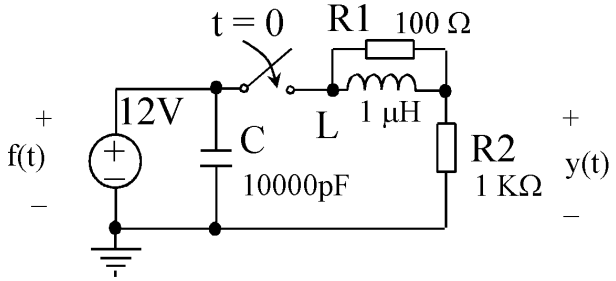

Figure 1:

6. **Text book problems:** 6.4-9 and 6.4-10.