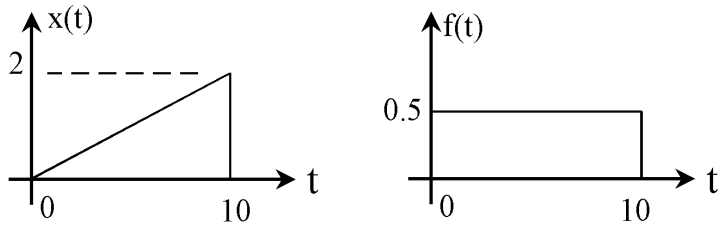
EE321 Fifth Homework Assignment

@ Kefu Xue, Ph.D., Sep. 2001

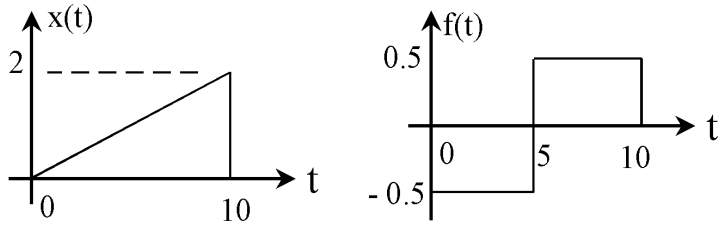
1 Measure the Similarity and Orthogonality of Signals

1. For the following pairs of signals, $x(t)$ and $f(t)$ for $t_0 \leq t \leq t_1$, find parameter c such that $x(t) = cf(t)$ with minimum error energy. Also calculate the energy of the error $e(t) = x(t) - cf(t)$.

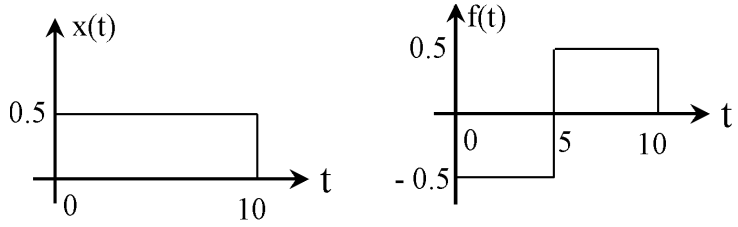
   (a) signals



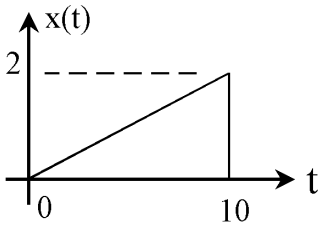
   (b) signals



   (c) signals



   (d) signals



and $f(t) = \sin(0.2\pi t)$ for $0 \leq t \leq 10$. Work out another case with $f(t) = \cos(0.2\pi t)$ for $0 \leq t \leq 10$. Verify your results using Matlab.

2. Show that if the following pairs of periodic signals, $x(t)$ and $f(t)$ are orthogonal or not.

   (a) $x(t) = \cos(\omega_0 t)$ and $f(t) = \sin(\omega_0 t)$

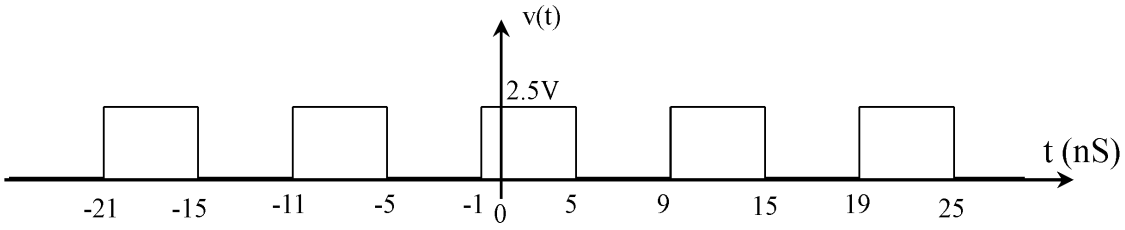
   (b) $x(t) = e^{j\omega_0 t}$ and $f(t) = e^{jn\omega_0 t}$ where $n \neq 1$

   (c) $x(t) = \cos(\omega_0 t)$ and $f(t) = 2\cos(\omega_0 t - 1)$

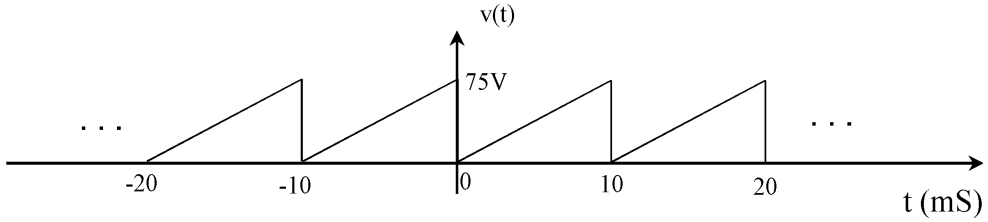
2 Fourier Series Representation of Periodic Signals

3. Find Exponential Fourier Series Representation of the following signals.

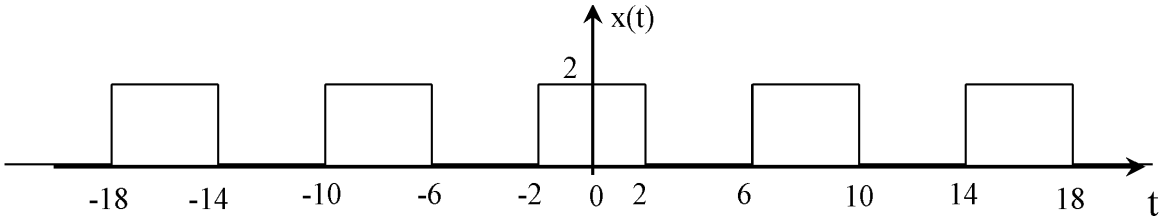
   (a) signal



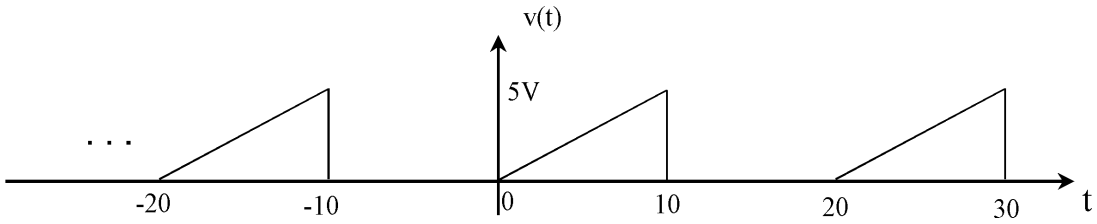
   (b) signal



   (c) signal



(d) signal



(e) signal $x(t) = 2\cos(200\pi t) - 3\cos(160\pi t - 1.2) + 2 - \sin(80\pi t)$, sketch the magnitude and phase Fourier series spectra and verify the result with Matlab fft() function.

(f) signal $x(t) = 1 + 2\cos(20\pi t) - 3\cos(16\pi t - 1) - \sin(8t)$

(g) signal $x(t) = \cos(27t) + 2\sin(6t - 1.2) - 3\sin(18t)$, sketch the magnitude and phase Fourier series spectra and verify the result with Matlab fft() function.

4. Given the following sets of exponential Fourier coefficients of real signals, find the signal expression in time domain and plot the signals in Matlab.

   (a) $\omega_0 = 10$ rad./sec., $D_0 = 2$, $D_2 = 2j$, $D_{-5} = 1 + j$

   (b) $\omega_0 = 6\pi$ rad./sec., $D_{-1} = 2$, $D_3 = 0.5e^{-j}$, $D_5 = 0.3 + 0.1j$, $D_{-7} = -0.1$

5. For the 4 graphical signals in problem 3, if only first 11 ($D_{-5}, D_{-4}, \cdots, D_0, D_1, \cdots, D_5$) Fourier series coefficients are used in their Fourier series representations, what are the mean square errors of the reduced Fourier series representations with respect to their original signal representations? (Hint: using Parseval's theorem) Plot all 4 synthesized signals using only the first 11 of their corresponding Fourier series coefficients in Matlab.

6. Given a full–wave rectified signal $x(t) = |12\cos(400\pi t)|$, find its exponential Fourier series representation.. Determine the number N such that the first $2N + 1$ (from D_{-N} to D_N) Fourier series coefficients cover more than 99.75% of the signal power.

7. **Extra exercise problems in the text book:**