EE322 First Midterm Exam (1)

1) $f(t) = 3\sin(240\pi t - \frac{\pi}{4})$ (V)

a) $\omega_0 = 240\pi$, $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{240\pi} = \frac{1}{120}$ (sec.)

$f_0 = \frac{\omega_0}{2\pi} = 120$ Hz or $f_0 = \frac{1}{T_0} = 120$ Hz

b) $3\sin(240\pi t - \frac{\pi}{4}) = 3\cos(240\pi t - \frac{\pi}{2} - \frac{\pi}{4})$

$= 3\cos(240\pi t - \frac{3\pi}{4}) = 3\cos(240\pi(t - \frac{3\pi}{4\cdot 240\pi}))$

Delay time $\tau_0 = \frac{3\pi}{4\cdot 240\pi} = \underline{\underline{\frac{1}{320}}}$ seconds

c) $P(t) = \frac{f^2(t)}{1\Omega} = f^2(t) = 9\sin^2(240\pi t - \frac{\pi}{4})$

$P_{av} = \frac{1}{T_0}\int_{T_0} P(t)\,dt = \frac{1}{T_0}\int_{T_0} 9\sin^2(240\pi t - \frac{\pi}{4})\,dt$

$= \frac{3^2}{2} = \frac{9}{2}$ (W)

RMS value $= \sqrt{P_{av}} = \frac{3}{\sqrt{2}}$ (V)

d) $P_{av\_100\Omega} = \frac{(\frac{3}{\sqrt{2}})^2}{100} = \frac{9}{200}$ (W)

$\because \frac{9}{200} < \frac{1}{16}$ $\therefore$ A 1/16 W resistor is sufficient for the applications.

c) If $x(t) = 10 f(t)$, then P_{av}^{x} for $x(t)$

is $(10)^2 \cdot \frac{9}{2}(W) = \frac{900}{2}(W)$

$$P_{av-100}^{x} = \frac{900}{200}(W) = 4.5W$$

The power rating of the resistor must be great than 4.5W.

2) a) $E_f = \int_{-\infty}^{\infty} f^2(t)\,dt = \int_1^5 t^2\,dt$

$$= \frac{t^3}{3}\Big|_1^5 = \frac{125}{3} - \frac{1}{3} = \frac{124}{3} \quad (\text{unit}^2 \cdot t)$$

b) $E_X = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} [5 \cdot f(2t+1)]^2\,dt$

$$= 5^2 \int_{-\infty}^{\infty} \Big[(2t+1)[u(2t+1-1) - u(2t+1-5)]\Big]^2 dt$$

$$= 25 \int_{-\infty}^{\infty} (2t+1)^2 [u(2t) - u(2t-2))]^2\,dt$$

$$= 25 \int_0^2 [(2t)^2 + 4t + 1]\,dt$$

$$= 25 \left[4\frac{t^3}{3}\Big|_0^2 + \frac{4t^2}{2}\Big|_0^2 + t\Big|_0^2\right]$$

$$= 25\left[\frac{32}{3} + 8 + 2\right] = \frac{800}{3} + 250 = 516.6667$$

$(\text{unit}^2 \cdot t)$

3)

$$x(t) = 2\left(\frac{e^{(3\pi t-1)j} - e^{-j(3\pi t-1)}}{2j}\right) + \frac{e^{j(5\pi t+0.5)} + e^{-j(5\pi t+0.5)}}{2}$$

$$= -je^{-j}e^{j3\pi t} + je^{j}e^{-j3\pi t} + \frac{1}{2}e^{j0.5}e^{j(5\pi t)} + \frac{1}{2}e^{-j0.5}e^{-j5\pi t}$$

$$= e^{-j\frac{\pi}{2}}e^{-j}e^{j3\pi t} + e^{j\frac{\pi}{2}}e^{j}e^{-j3\pi t} + \frac{1}{2}e^{j0.5}e^{j5\pi t} + \frac{1}{2}e^{-j0.5}e^{-j5\pi t}$$

$$\boxed{= e^{-j(\frac{\pi}{2}+1)}e^{j3\pi t} + e^{j(\frac{\pi}{2}+1)}e^{-j3\pi t} + \frac{1}{2}e^{j0.5}e^{j5\pi t} + \frac{1}{2}e^{-j0.5}e^{-j5\pi t}}$$ #

4) a) $\int_0^2 \cancel{\delta(t+1)\cos\left(\frac{\pi}{4}t\right)dt}^{\,0} + \int_0^2 \delta(t-1)\cos\left(\frac{\pi}{4}t\right)dt$

$\because \delta(t+1) = 1$ when $t=-1$

$$= \int_0^2 \delta(t-1)\cos\left(\frac{\pi}{4}t\right)dt = \cos\left(\frac{\pi}{4}\right)$$

b) $2e^{2x}\cdot\delta(x-0.5) = 2e^{2\cdot 0.5}\delta(x-0.5)$

$$= 2e\,\delta(x-0.5)$$

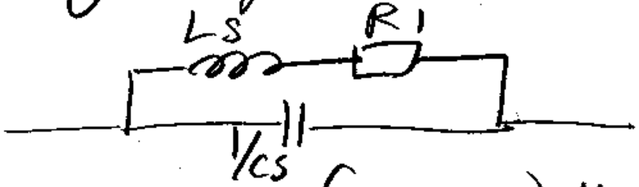
c) $\int_0^\infty 2\omega\,\delta(\omega-5\pi)\,e^{-j\omega}\,d\omega$

$= 2\cdot(5\pi)e^{-j5\pi} = 10\pi\,e^{-j\pi} = -10\pi$

d) $\left(t^2+2e^{2t}\right)\delta(t+1) = \left((-1)^2+2e^{-2}\right)\delta(t+1)$

$= \left(1+2e^{-2}\right)\delta(t+1)$

5) Using Impedance method.



$$Z_1(s) = (LS+R_1)\,/\!/\,\frac{1}{CS} = \frac{(LS+R_1)\cdot\frac{1}{CS}}{LS+R_1+\frac{1}{CS}}$$

$$= \frac{LS+R_1}{LCS^2+CR_1S+1}$$

Voltage Divider : $f(t)\cdot\left(\frac{R_2}{Z_1(s)+R_2}\right) = y(t)$

$$H(S) = \frac{R_2}{\frac{LS+R_1}{LCS^2+CR_1S+1}+R_2} = \frac{R_2(LCS^2+CR_1S+1)}{LS+R_1+R_2LCS^2+CR_1R_2S+R_2}$$

$$= \frac{LCR_2S^2+CR_1R_2S+R_2}{R_2LCS^2+(L+CR_1R_2)S+R_1+R_2}$$

$$= \frac{S^2+\frac{R_1}{L}S+\frac{1}{LC}}{S^2+\frac{L+CR_1R_2}{R_2LC}S+\frac{R_1+R_2}{R_2LC}}$$

$$= \frac{s^2 + 10^7 s + 10^{15}}{s^2 + 1.1\times 10^8 s + 1.01\times 10^{15}}$$

$B(s) = s^2 + 10^7 s + 10^{15}$

$A(s) = s^2 + 1.1\times 10^7 s + 1.01\times 10^{15}$

D.E.Q.

$$\frac{d^2 y(t)}{dt^2} + 1.1\times 10^7 \frac{dy(t)}{dt} + 1.01\times 10^{15} y(t)$$

$$= \frac{d^2 f(t)}{dt^2} + 10^7 \frac{d f(t)}{dt} + 10^{15}$$

Let $A(s) = 0$ find roots: $P_{1,2} = -0.55\times 10^7 \pm j\,3.13\times 10^7$

$\because \mathrm{Re}\{P_{1,2}\} = -0.55\times 10^7 < 0$

The poles on the LHS.

The system is stable.

6) $$M\frac{d^2 x(t)}{dt^2} + K_d \frac{dx(t)}{dt} + K_s x(t) = f(t)$$

$A(s) = Ms^2 + K_d s + K_s$; $B(s) = 1$

or $A(s) = s^2 + \frac{K_d}{M} s + \frac{K_s}{M}$; $B(s) = \frac{1}{M}$

$$H(s) = \frac{B(s)}{A(s)} = \frac{1}{Ms^2 + K_d s + K_s}$$

(7) $y(t) = h(t) * f(t) = \int_{-\infty}^{\infty} h(t-\tau) f(\tau) d\tau$

$$= \int_{-\infty}^{\infty} e^{-6(t-\tau)} u(t-\tau-1) \cdot 2e^{-2(\tau+1)} u(\tau-3) d\tau$$

$$= \int_{3}^{t-1} \underline{e^{-6t}} e^{6\tau} \cdot \underline{2} e^{-2\tau} \underline{e^{-2}} d\tau$$

$$= 2e^{-2} e^{-6t} \int_{3}^{t-1} e^{4\tau} d\tau$$

$$= 2e^{-2} e^{-6t} \left(\frac{1}{4} e^{4\tau} \Big|_{3}^{t-1} \right)$$

$$= 2e^{-2} e^{-6t} \left(\frac{1}{4} e^{4(t-1)} - \frac{1}{4} e^{12} \right)$$

$$= \frac{1}{2} e^{-2} e^{-4} e^{-2t} - \frac{1}{2} e^{-2} e^{-6t} e^{12}$$

$$= \frac{1}{2} e^{-6} \cdot e^{-2t} - \frac{1}{2} e^{10} e^{-6t} \quad \underline{4 \le t}$$

$\because S_h = 1, \ S_f = 3, \ \therefore S_y = 1 + 3 = 4$

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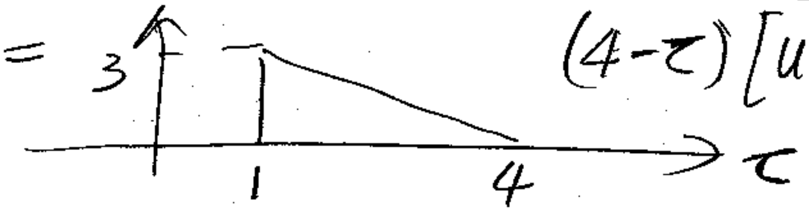
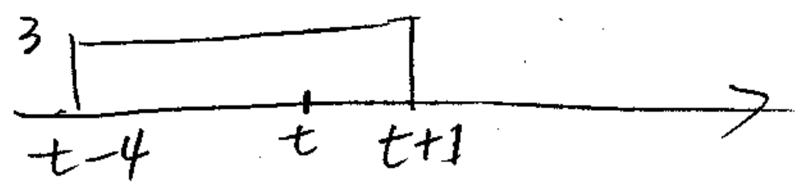
8) $h(\tau) =$  $(4-\tau)\left[u(\tau-1) - u(\tau-4)\right]$

$x(t-\tau)$



$S_h = 1$	$E_h = 4$
$S_x = -1$	$E_x = 4$
$S_y = 0$	$E_y = 8$

$t+1 \geq 4$ leads $t \geq 3$

$t-4 \geq 1$ leads $t \geq 5$

$$y(t) = \int_1^{t+1} 3(4-\tau)\,d\tau \qquad \boxed{0 \le t < 3}$$

$$= \frac{-3(4-\tau)^2}{2}\Bigg|_1^{t+1} = -\frac{3}{2}(-t+3)^2 - \frac{27}{2}$$

$$= -\frac{3}{2}(t^2 - 6t + 9) - \frac{27}{2} = \underline{-\frac{3}{2}t^2 + 9t}$$

$$y(t) = \int_1^{4} 3(4-\tau)\,d\tau \qquad \boxed{3 \le t < 5}$$

$$= -\frac{3(4-\tau)^2}{2}\Bigg|_1^{4} = \underline{\frac{27}{2}}$$

$$y(t) = \int_{t-4}^{4} 3(4-\tau)\,d\tau \qquad \boxed{5 \le t \le 8}$$

$$= \frac{-3(4-\tau)^2}{2}\Bigg|_{t-4}^{4} = \frac{3}{2}(4-t+4)^2$$

$$= \underline{\frac{3}{2}(8-t)^2}$$

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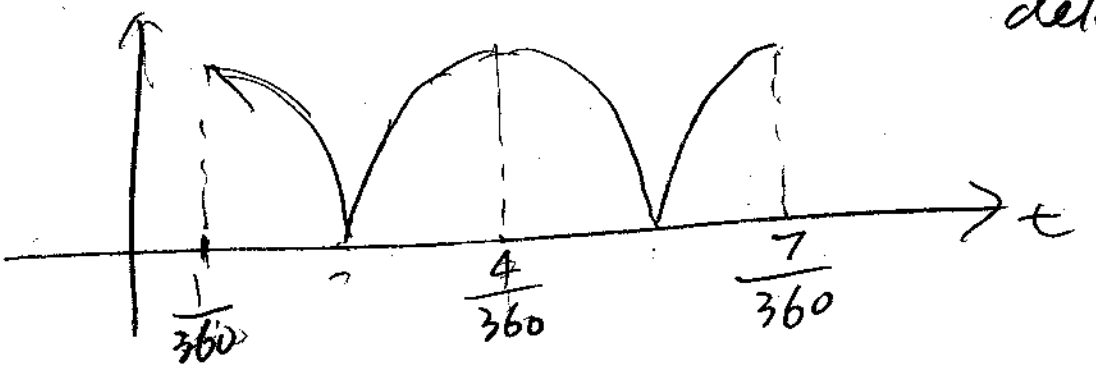
9)(a) $i(t) = \left|4\cos\left(120\pi t - \frac{\pi}{3}\right)\right|$ period $= \frac{1}{60}$ sec.

delay $= \frac{\pi}{3} \Big/ 120\pi$

$= \frac{1}{360}$ sec.



(8)

b) $\because i^2(t) = \left|4\cos(120\pi t - \frac{\pi}{2})\right|^2$

$= \left(4\cos(120\pi t - \frac{\pi}{2})\right)^2 \quad (A^2)$

$\therefore$ RMS value is the same as Sinusoidal function $= \frac{4}{\sqrt{2}}$ (A)

c) $P_{av\_10\Omega} = \left(\frac{4}{\sqrt{2}}\right)^2 \cdot 10 = 80\ W$

(10) $E_f = \int_{-\infty}^{\infty} f^2(t)\, dt$

$E_{f(t\pm t_0)} = \int_{-\infty}^{\infty} f^2(t\pm t_0)\, dt = \int_{-\infty}^{\infty} f^2(x)\, dx = E_f$

let $x = t\pm t_0$

$dx = dt$

$E_{f(at)} = \int_{-\infty}^{\infty} f^2(at)\, dt = \int_{-\infty}^{\infty} f^2(x)\, \frac{dx}{a}$

let $x = at$

$dx = a\,dt$

$= \frac{1}{a}\int_{-\infty}^{\infty} f^2(x)\, dx$

$= \frac{E_f}{a}$

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