BE 321 Second Midterm Exam ①

1 a) $f(t) = (10 - e^{-5t})u(t-2) = 10\,u(t-2) - e^{-10}e^{-5(t-2)}u(t-2)$

$F(s) = \frac{10}{s}e^{-2s} - \frac{e^{-10}e^{-2s}}{s+5}$

b) $x(t) = (\cos(2\pi t) - 3)^3\delta(t) = (1-3)^3\delta(t) = -8\,\delta(t)$

$X(s) = -8$

c) $y(t) = e^{-7t}u(t) * 3e^{-3t}u(t)$

$Y(s) = \frac{1}{s+7}\cdot\frac{3}{s+3} = \frac{3}{(s+7)(s+3)}$

d) $w(t) = 2t\,u(t-2) = (2(t-2)+4)u(t-2)$

$W(s) = \frac{2}{s^2}e^{-2s} + \frac{4}{s}e^{-2s}$

2. a) $F_1(s) = \frac{s+6}{(s+1)(s+2)(s+4)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+4}$

$A = \left.\frac{s+6}{(s+2)(s+4)}\right|_{s=-1} = \frac{5}{3}$; $B = \left.\frac{s+6}{(s+1)(s+4)}\right|_{s=-2} = \frac{4}{-2} = -2$

$C = \left.\frac{s+6}{(s+1)(s+2)}\right|_{s=-4} = \frac{2}{-3\cdot(-2)} = \frac{1}{3}$

$f_1(t) = \left(\frac{5}{3}e^{-t} - 2e^{-2t} + \frac{1}{3}e^{-4t}\right)u(t)$

b) $F_2(s) = \frac{(s-5)e^{-2s}}{(s+2)(s+3)} = \left(\frac{A}{s+2} + \frac{B}{s+3}\right)e^{-2s}$

$A = \left.\frac{s-5}{s+3}\right|_{s=-2} = -7$; $B = \left.\frac{s-5}{s+2}\right|_{s=-3} = 8$

$f_2(t) = \left(-7e^{-2(t-2)} + 8e^{-3(t-2)}\right)u(t-2)$

c) $F_3(s) = \frac{s}{(s+1)(s+1-2j)(s+1+2j)} = \frac{A}{s+1} + \frac{C}{s+1-2j} + \frac{C^*}{s+1+2j}$

Continue 2 c).

$$A = \left. \frac{s}{s^2+2s+5} \right|_{s=-1} = \frac{-1}{1-2+5} = -\frac{1}{4}$$

$$C = \left. \frac{s}{(s+1)(s+1+2j)} \right|_{s=-1+2j} = \frac{-1+2j}{2j(4j)} = \frac{-1+2j}{-8} = \frac{1}{8} - \frac{1}{4}j$$

$$= 0.2795e^{-j1.1071}$$

$$f_3(t) = \left[-\frac{1}{4}e^{-t} + 2 \cdot 0.2795e^{-t}\cos(2t-1.1071)\right]u(t)$$

$$= \left[-\frac{1}{4}e^{-t} + 0.559e^{-t}\cos(2t-1.1071)\right]u(t)$$

3. a) $F_1(s) = \frac{s^2+s+2}{(s+2)(s^2+4s+3)} = \frac{s^2+s+2}{(s+2)(s+1)(s+3)}$ is stable

Since all of its poles are on the L.H.S.

Final value: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF_1(s) = \lim_{s\to 0} \frac{s^3+s^2+2s}{s^3+6s^2+11s+6} = 0$

Initial value: $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF_1(s) = \lim_{s\to\infty} \frac{s^3+s^2+2s}{s^3+6s^2+11s+6}$

$$= \lim_{s\to\infty} \frac{1+\frac{1}{s}+\frac{2}{s^2}}{1+\frac{6}{s}+\frac{11}{s^2}+\frac{6}{s^3}} = 1$$

b) $F_2(s) = \frac{6s}{s^2+5s-6} = \frac{6s}{(s-2)(s-3)}$ is not stable

Since not all the poles are on the LHS.

Final value does not exist.

Initial value: $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF_1(s) = \lim_{s\to\infty} \frac{6s^2}{s^2+5s-6}$

$$= \lim_{s\to\infty} \frac{6}{1+\frac{5}{s}-\frac{6}{s^2}} = 6$$

4) a) $f_1(t) = 2 + \cos(2t+2)$ the frequencies of of the eigenfunction is $\omega = 0$ and $\omega = 2$

$\underline{H(j0) = \frac{1}{2}}$, $\underline{H(j2)} = \frac{(j2)^2+4}{(j2)^2+6(j2)+8} = \frac{-4+4}{-4+8+12j}$

eigen values. $= 0$

$y_{ss}(t) = \frac{1}{2} \cdot 2 + 0 \cdot \cos(2t+2) = 1$

b). $f_2(t) = 2\cos(10\pi t - 1)$, frequency is equal to $\omega = 10\pi$, eigen value $H(j10\pi) = \frac{(j10\pi)^2+4}{(j10\pi)^2+6 \cdot j10\pi+8}$

$H(j10\pi) = \frac{-100\pi^2+4}{-100\pi^2+8+j60\pi} = 0.9682 + j0.1864$

$= 0.986\, e^{j0.1902}$

$y_{ss}(t) = 2 \cdot 0.986 \cos(10\pi t - 1 + 0.1902)$
$= 1.972 \cos(10\pi t - 0.8098)$

5). $A(s) = s^2+9s+14$, $B(s) = 2s+1$

a) $H(s) = \frac{B(s)}{A(s)} = \frac{2s+1}{s^2+9s+14}$

b) From A(s)
$s^2 Y(s) - s\, y(0^-) - \dot{y}(0^-) + 9[s\, Y(s) - y(0^-)] + 14\, Y(s)$

From B(s)
$2[s\, X(s) - x(0^-)] + X(s)$

$y(0^-) = 2$
$\dot{y}(0^-) = -1$
$x(0^-) = 0$

$Y(s) = \frac{(2s+1)\, X(s)}{(s^2+9s+14)} + \frac{2s-1+18}{s^2+9s+14}$

zero input response $(X(s)=0)$

$$Y(s) = \frac{2s+17}{(s+2)(s+7)} = \frac{A}{s+2} + \frac{B}{s+7}$$

$$A = \left.\frac{2s+17}{s+7}\right|_{s=-2} = \frac{13}{5} \; ; \; B = \left.\frac{2s+17}{s+2}\right|_{s=-7} = \frac{-3}{5}$$

$$y_{int}(t) = \left(\frac{13}{5} e^{-2t} - \frac{3}{5} e^{-7t}\right) u(t)$$

c) zero state response $X(s) = 10 \frac{1}{s+4}$

$$Y(s) = \frac{(2s+1)\cdot 10}{(s+2)(s+7)(s+4)} = \frac{A}{s+2} + \frac{B}{s+7} + \frac{C}{s+4}$$

$$A = \left.\frac{10(2s+1)}{(s+7)(s+4)}\right|_{s=-2} = \frac{10(-3)}{5 \cdot 2} = -3$$

$$B = \left.\frac{10(2s+1)}{(s+2)(s+4)}\right|_{s=-7} = \frac{\cancel{10}^{2}(-13)}{-\cancel{5} \cdot -3} = \frac{-26}{3}$$

$$C = \left.\frac{10(2s+1)}{(s+2)(s+7)}\right|_{s=-4} = \frac{\cancel{10}^{5}(-7)}{-\cancel{2} \cdot 3} = \frac{35}{3}$$

$$y_{zs}(t) = \left(-3 e^{-2t} - \frac{26}{3} e^{-7t} + \frac{35}{3} e^{-4t}\right) u(t)$$

d) $y_{total}(t) = y_{int}(t) + y_{zs}(t)$

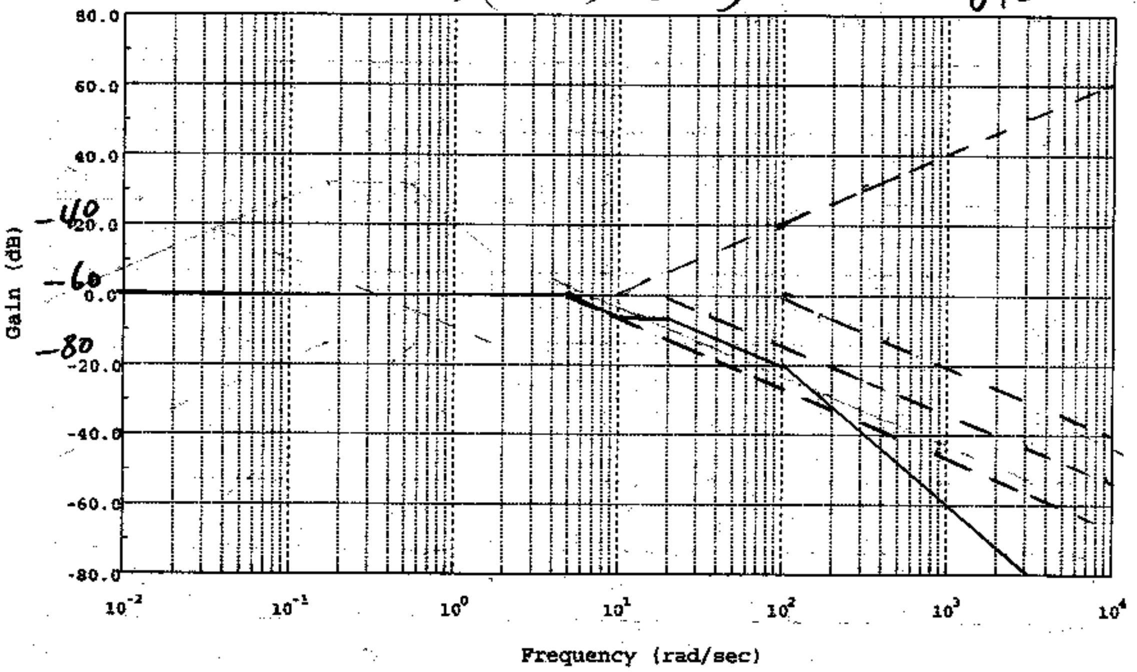
$$= \left[\left(\frac{13}{5} - 3\right) e^{-2t} - \left(\frac{3}{5} + \frac{26}{3}\right) e^{-7t} + \frac{35}{3} e^{-4t}\right) u(t)$$

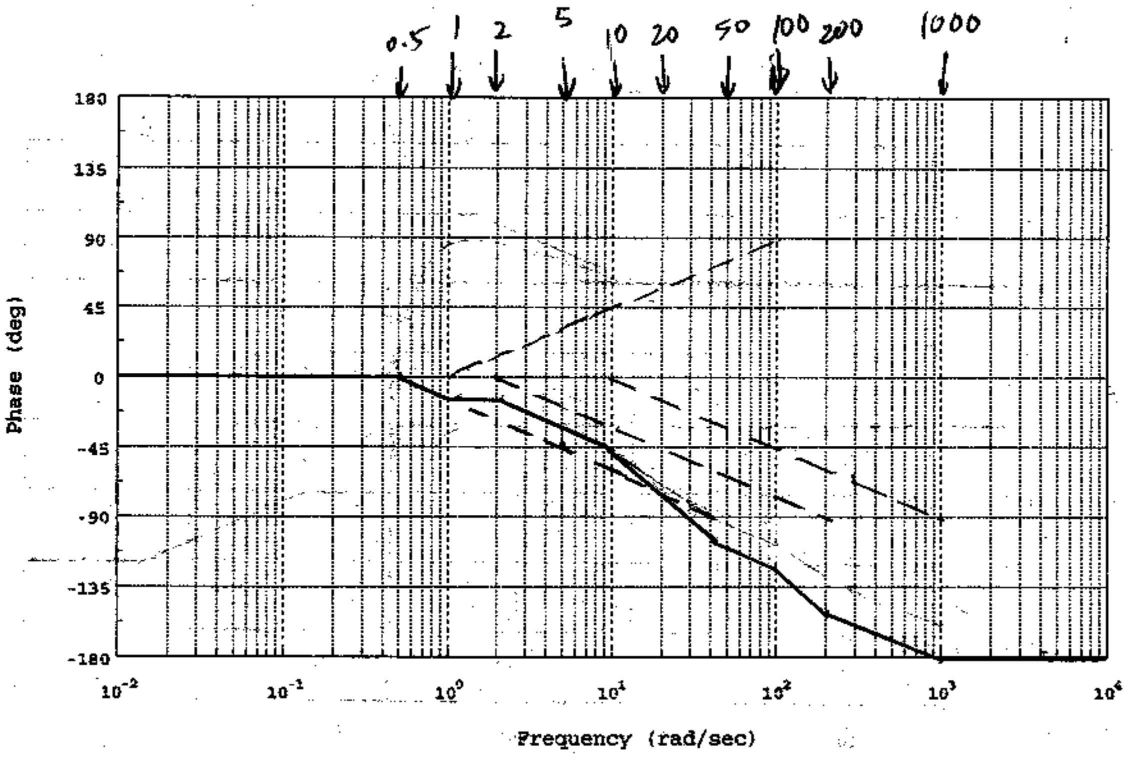
$$= \left(-\frac{2}{5} e^{-2t} - \frac{139}{15} e^{-7t} + \frac{35}{3} e^{-4t}\right) u(t).$$

$$H_1(s) = \frac{s+10}{(s+100)(s+20)(s+5)} = \frac{10\left(\frac{s}{10}+1\right)}{100\cdot 20\cdot 5\left(\frac{s}{100}+1\right)\left(\frac{s}{20}+1\right)\left(\frac{s}{5}+1\right)}$$

$$= \frac{1}{1000}\,\frac{\left(\frac{s}{10}+1\right)}{\left(\frac{s}{100}+1\right)\left(\frac{s}{20}+1\right)\left(\frac{s}{5}+1\right)}$$

$$20\log_{10}\frac{1}{1000} = -60\,dB$$





$$H_2(s) = \frac{10^4 \cdot 20 s\left(\frac{s}{20}+1\right)}{10 \cdot 200 \cdot 3600\left(\frac{s}{200}+1\right)\left(\frac{s^2}{3600}+\frac{85s}{3600}+1\right)}$$

complex roots

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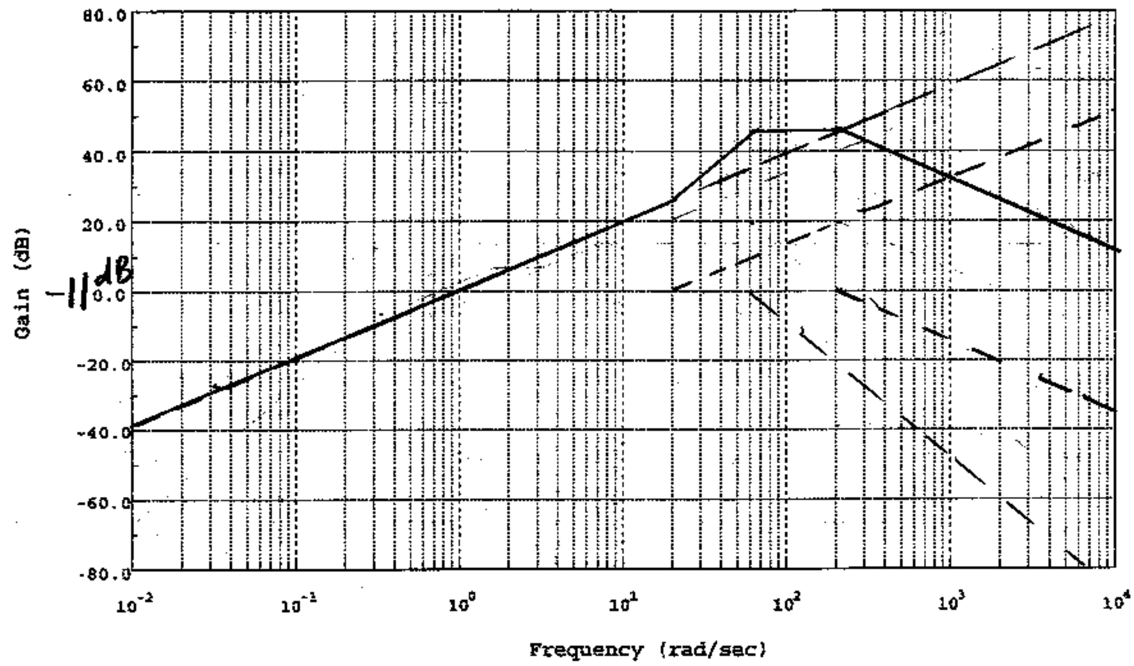
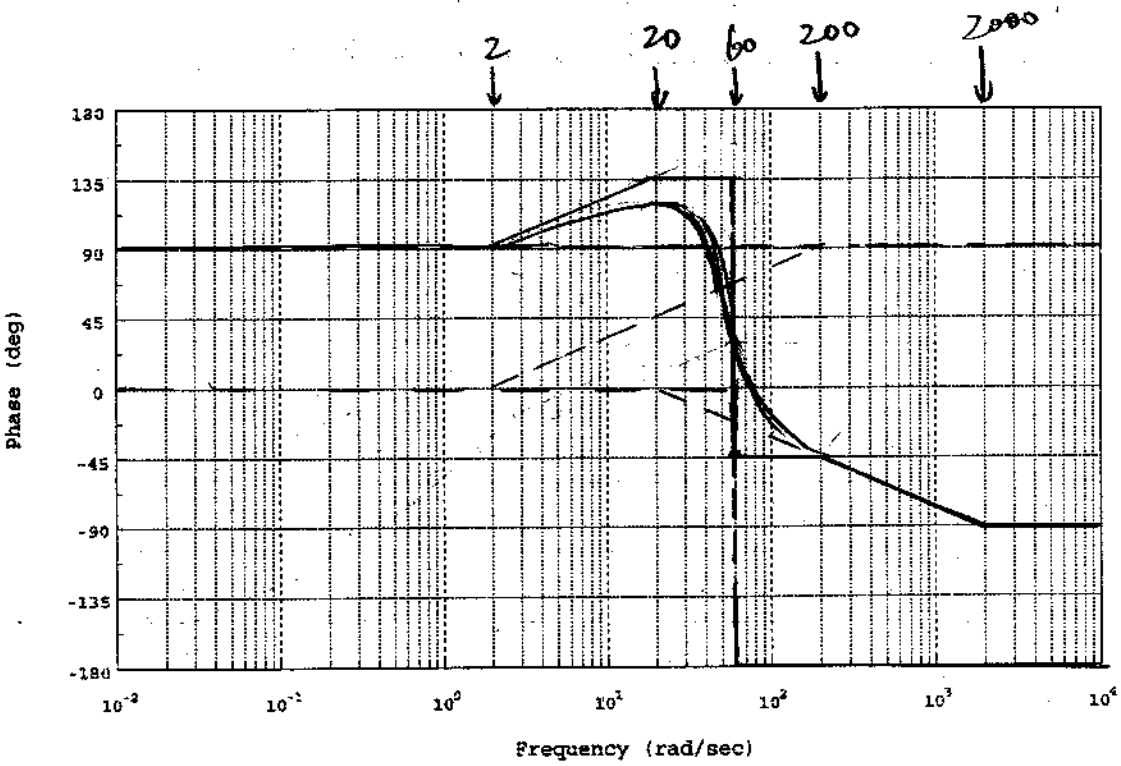
$$20\log_{10}\left(\frac{10}{36}\right) = 20\log_{10}10 - 40\log_{10}6 = -11\text{ dB}$$

$$\omega_n^2 = 3600,\quad \omega_n = 60$$





7a) - $\frac{1}{Cs} \parallel R_2 = \dfrac{\frac{1}{Cs}\cdot R_2}{\frac{1}{Cs}+R_2} = \dfrac{R_2}{1+R_2Cs}$

Voltage divider:

$$H(s) = \dfrac{\frac{R_2}{1+R_2Cs}}{R_1 + \frac{R_2}{1+R_2Cs}} = \dfrac{R_2}{R_1R_2Cs + R_1 + R_2}$$

$$= \dfrac{R_2}{R_1+R_2}\,\dfrac{1}{\dfrac{s}{\left(\frac{R_1+R_2}{R_1R_2C}\right)}+1}$$

$$\omega_c = \dfrac{R_1+R_2}{R_1R_2C} = \dfrac{10100}{10^2\,10^6\times10\times10^{-6}} = \dfrac{10100}{10} = 1010$$

b) $H(j0) = \dfrac{R_2}{R_1+R_2} = 0.9901$

$H(j\omega_c) = H(j1010) = 0.9901\,\dfrac{1}{j+1} = 0.9901\cdot\frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}$

$= 0.7e^{-j\frac{\pi}{4}}$

$H(j10\omega_c) = H(j10100) = 0.9901\,\dfrac{1}{j10+1} = 0.9901\,\dfrac{1}{10.05}e^{-j1.4711}$

$= \boxed{0.0905e^{-j1.4711}}$

eigenvalue amplitude changes from 0.9901, 0.7, to 0.0905 as ω increases. It's a Low pass filter.

c) $C = 50\,\mu F$ $\quad \omega_c = \dfrac{R_1+R_2}{R_1R_2C} = \dfrac{10100}{10^2\cdot10^6\cdot50\times10^{-6}} = \dfrac{10100}{50}$

$= \dfrac{1010}{5} = 202$

The cut off frequency reduces by 5 times.