EE322 First Homework Problem

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1. Given an analog signal $x_a(t) = 2\sin(250\pi t + \frac{\pi}{3}) + \cos(550\pi t - \frac{\pi}{3}) + 2$

   (a) Find the fundamental frequency ω_0 and Fourier series representation and its co-efficients X_k using Fourier series recognition method.

   (b) Select a sampling frequency which is 100 Hz higher than the Nyquist Frequency (twice of the highest frequency component) of the signal $x_a(t)$ (in Hz).

   (c) Is there an aliasing error in this sampling process? Why?

   (d) Find the expression for the sampled sequence $x(n)$.

   (e) Is the sequence $x(n)$ periodic? If yes, find the minimum period (in number of samples) of the sequence. If not, explain why.

   (f) Find DFS coefficients using Matlab^{TM}. Matching the Matlab^{TM} result with your hand calculated result.

2. For the same analog signal in the previous problem (1), select the sampling frequency which is 4 times of the Nyquist frequency (4 times over sampling) for the expression of $x(n)$.

   (a) What is the minimum period (in number of samples) of the sequence $x(n)$ at this time?

   (b) Find DFS coefficients using Matlab^{TM}. Matching the Matlab^{TM} result with your hand calculated result.

3. For the same analog signal in problem 1, if we take 3 samples in each period of the highest frequency component in $x_a(t)$ to form the expression of $x(n)$, what is the sampling frequency? Is there an aliasing error in this sampling process? Why? What is the period of $x(n)$ in this case?

4. Compare an analog signal $w_a(t) = 2\cos(250\pi t) + \cos(550\pi t) + 1$ with the signal in problem 1.

   (a) Find the Fourier series representation and its coefficients X_k of the new signal using Fourier series recognition method. What are the differences in their coefficients?

   (b) In Matlab^{TM}, plot 3 period ($3N$) of $w(n)$ and $x(n)$ sampled with the 4 times over sampling frequency. What is the time duration (in second) of the plotted signals?

   (c) Plot each frequency components of $w(n)$ and $x(n)$ in separate plots with the same length ($3N$). From observation, comments on the following questions:

      i. How many periods of analog signals $w_a(t)$ and $x_a(t)$ in $3N$ samples of $w(n)$ and $x(n)$? Do your analysis to see if it matches your calculation.

ii. How many periods of each frequency components do you see in the plots? Show your analysis to match your observation.

iii. Comment on the wave forms of signals $w_a(t)$ and $x_a(t)$ and their relationship with their signal components (time/phase delay and amplitude).

5. Given the FS coefficients: $C_0 = 0.5$, $C_1 = 3e^{-j}$, $C_3 = -j$, and $C_5 = e^{j\frac{\pi}{6}}$ of a real-valued periodic signal, $x_a(t)$, with a period equal to $2 \cdot 10^{-3}$ seconds, find the expression of the signal.

6. A middle C piano note (262 Hz) will be sent by a AM traffic radio station (550 KHz) in the morning 4am as the test signal. The signal expression is shown as follows.

$$[2 + \sin(262 \cdot 2\pi t - 1)] \cos(550 \cdot 10^3 \cdot 2\pi t)$$

Find and sketch the line spectrum of the radio station signal. What is the Nyquist Frequency of the signal?

7. Consider the following analog signal :

$$x(t) = sin(120\pi t) + \frac{sin(360\pi t)}{3} + \frac{sin(600\pi t)}{5} + \frac{sin(840\pi t)}{7} + \frac{sin(1080\pi t)}{9}$$

which is a square-wave with signal components up to 9th harmonic and a fundamental frequency of 120π radian/second.

(a) Select the sampling frequency that is twice the Nyquist frequency. The sampling duration should include 4 periods of the square-wave. Generate all of the harmonic components and add them together to form the composite signal. Generate hard copies of the plots for the waveforms with up to 5th, 7th and 9th harmonic contents (3 plots), respectively. Observe the changes in the waveform on the screen as each harmonic content is being added into the signal.

(b) Change the sampling frequency to 750 Hz and sample 4 periods of the signal using this new sampling frequency. Generate hard copies of the plots for the waveforms with up to 5th, 7th and 9th harmonic contents (3 plots), respectively.

(c) Compare and contrast these six plots and discuss your observation and conclusions in a few sentences.

(d) Just for fun, you can include different delays in the harmonic components and then add them together to see the changes in signal waveforms. This is so called harmonic distortion.

8. Text book problems: 8.2-4, 8.2-6, 8.3-4, 8.4-1 and 8.4-3(a,b,c).

Tips:

- Use a text editor to put all the Matlab^{TM} commands in one file (yourfile.m) referred to as m-file and run the m-file in Matlab^{TM}. It will save your time.

- Use Matlab "subplot()" command to put multiple plots in one figure. It will reduce the number of pages to plot.

For example, let wn and xn be two signals in problem 1 and wn1, wn2, wn3, xn1, xn2, xn3 are their signal components. We can put their plots in one figure using the following code.

```
% subplot examples
% The subplot create 4 rows and 2 columns of plots in one page.
subplot(4,2,1);plot(t,xn);title('original signal xn'); %in 1st row 1st column
subplot(4,2,2);plot(t,wn);title('new signal wn');%in 1st row 2nd column
subplot(4,2,3);plot(t,xn1);title('1st component of xn'); %in 2nd row 1st column
subplot(4,2,4);plot(t,wn1);title('1st component of wn'); %in 2nd row 2nd column
subplot(4,2,5);plot(t,xn2);title('2st component of xn'); %in 3rd row 1st column
.....
```

This code will create 8 plots on one figure in the following format.

xn	wn
xn1	wn1
xn2	wn2
xn3	wn3