EE322 Second Homework Problems

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1. A LTI system is described by the following impulse response

$$h(n) = 2^{-n} \quad \text{for } n = -2, -1, \cdots, 3, 4$$

For a given input signal

$$\left\{ \begin{array}{|r|r|c|c|} \hline x(n) = \ 1 & -1 & 1 & 2 \\ n = \ 3 & 4 & 5 & 6 \\ \hline \end{array} \right\},$$

answer the following questions.

(a) Find the difference equation representation of the filter and system transfer function $B(z)$.

(b) What is the length of the zero–state response sequence $y(n)$?

(c) What are the beginning and ending indices of the zero–state response $y(n)$?

(d) Find some of the values of the zero–state response sequence

$$y(-2), y(0), y(4), y(8), y(11)$$

using the linear convolution sum.

(e) Verify your result using Matlab^{TM} function conv().

2. The unit impulse response of a linear system is $h(n) = [-1 \ 2 \ -1]$ for $n = 0, 1, 2$. The input signal $x(n) = n^2 \cdot [u(n+1) - u(n-4)]$, use the linear convolution sum to find the zero-state output signal $y(n)$.

(a) Find start index, S_y, ending index, E_y and length, L_y of the output signal.

(b) Show the linear convolution calculation and list the output signal $y(n)$ in a table with correct time index.

3. A digital filter is described by the following difference equation

$$y(n) - 0.9y(n-1) = x(n) + x(n-1).$$

(a) Find the polynomials, $A(z)$ and $B(z)$, and the transfer function $H(z)$.

(b) Given the initial condition $y(2) = 1$, find first 4 output values of the filter to the input signal $x(n) = 0.5^n u(n-1)$ using the difference equation iteratively.

4. A digital filter has the following transfer function

$$H(z) = \frac{1.1 - z^{-1}}{1 + 0.9z^{-2}}$$

(a) Find the expression of difference equation.

(b) Find the first 4 output values of the filter to the input signal $x(n) = u(n-1)$ iteratively with initial conditions: $y(1) = 0.5$ and $y(0) = 0$.

5. Given the difference equation of a digital filter

$$y(n) = x(n) + 2x(n-1) + x(n-1),$$

find its unit impulse response sequence, $h(n)$, and transfer function, $H(z)$. Any conclusion?

6. A linear time–invariant continuous–time integrator is described by the following differential equation.

$$y_a(t) = \frac{d^2x_a(t)}{dt^2} - 5\frac{dx_a(t)}{dt} + x_a(t)$$

Find the corresponding difference equation with input $x(n)$, output $y(n)$ and $B(z)$ using the approximation,

$$\frac{dx_a(t)}{dt} \approx \frac{x_a(nT) - x_a(nT - T)}{T} = \frac{x(n) - x(n-1)}{T}.$$

Hint: $\frac{d^2x_a(t)}{dt^2} = \frac{d}{dt}\{\frac{dx_a(t)}{dt}\}$

7. Consider the following 4 signals:

$$\begin{aligned}
w_a(t) &= \cos(2\pi t)[u(t) - u(t-1)] \\
x_a(t) &= \sin(2\pi t)[u(t) - u(t-1)] \\
y_a(t) &= 2t \cdot [u(t) - u(t-1)] \\
z_a(t) &= 2(1-t) \cdot [u(t) - u(t-1)]
\end{aligned}$$

(a) Find the Energy ($\gamma_{xx}(0)$) of each signal.

(b) Calculate $\gamma_{xy}(0)$, $\gamma_{xz}(0)$, $\gamma_{wy}(0)$, and $\gamma_{wz}(0)$ using integral and explain the implications of the calculation results.

(c) Verify the above results using Matlab (The functions may be involved: sum(), .*, .^). (using $T = 0.001$ s)

(d) Using Matlab statement: fgamaxz=fliplr(conv(fliplr(xa), za)); to calculate the correlation functions: $\gamma_{xy}(k)$, $\gamma_{xz}(k)$, $\gamma_{wy}(k)$, and $\gamma_{wz}(k)$. Plot and interpret the correlation functions. (Matlab function may be involved: [maxv,maxi]=max(fgamaxz);)