EE322 3rd Homework Assignment

@ Kefu Xue, Ph.D., Sep. 2001 to July 2007

Instructor: Kefu Xue, Ph.D.

1. Given the frequency response of an ideal lowpass filter:

$$H(e^{j\theta}) = \begin{cases} 1 & |\theta| \le \theta_c \\ 0 & \theta_c < |\theta| \le \pi \end{cases}$$

   (a) Using the definition of inverse DtFT to show that the unit impulse response, $h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\theta})e^{j\theta n}d\theta$ is a sinc() function of n.

   (b) Sketch both the ideal lowpass filter spectrum $H(e^{j\theta})$ and the unit impulse response $h(n)$ with $\theta_c = \frac{\pi}{4}$.

2. A digital signal processing (DSP) board equipped with an analog to digital converter (ADC) which has five adjustable sampling frequencies: 6 KHz, 8 KHz, 16 KHz, 44.1 KHz, 48 KHz. The DSP board is used for DFT analysis of various length up to $N = 1024$.

   (a) What is the highest frequency components in an analog signal can be sampled without aliasing error by this DSP board?

   (b) If an analog signal to be analyzed by the DSP board has a bandwidth of 10 KHz, what is you choice of sampling frequency such that the best (finest) frequency resolution can be achieved in the DFT analysis and what is the frequnecy resolution (Hz)?

   (c) If a sampling frequency $f_s = 16$ KHz and length $N = 1024$ are selected, which frequencies do 32nd, 500rd, and 622nd samples in the DFT sequence represent?

   (d) If the sampling frequency of the DSP board is selected to 6 KHz and a 256 points DFT is performed on an analog signal, what is the record length (duration) of the analog signal collected for DFT (in second)? To the same data, in order to achieve a frequency resolution better than 10 Hz, how many zeros have to be padded?

3. Given two samples of the DFT of a real valued sequence $x(n)$, $\quad n = 0, 1, 2, 3$.

$$X(0) = 4, \quad X(2) = -3$$

   which one of the following sequences be the possible $x(n)$? Find the DFT values of $X(1)$ and $X(3)$ of that sequence.

   (a) $\{\ 4 \ \ 1.5 \ \ -3 \ \ 2 \ \ -0.5\ \}$

   (b) $\{\ 7 \ \ 2 \ \ -6.5 \ \ 1.5\ \}$

(c) $\{\ 4 \quad -1 \quad -3 \quad 4\ \}$
(d) $\{\ -3 \quad 5 \quad 3.5 \quad -1.5\ \}$

4. Part of a 4 point DFT, $X(k)$, of a real valued signal sequence is given:

$$X(0) = 2; \quad X(2) = 0; \quad X(3) = 1 - 2j$$

   (a) If $X(2)$ represent the frequency component of 50 Hz, what is the sampling frequency that is used in sampling signal $x_a(t)$? (Explain)
   (b) Find the values of $x(n), \quad n = 0, 1, 2, 3.$

5. Some values of the DFT sequence $X(k)$ of a 20 points real sequence $x(n)$ zero-padded to the nearest power of 2 are known as follows. $X(0) = 20, X(3) = -3 + 2j, X(4) = 5, X(17) = -4 - 3j, X(26) = -1 + 4j$, and $X(25) = j$.

   (a) What is the length of the DFT sequence $X(k)$?
   (b) What is the average value (DC) of the real sequence $x(n)$?
   (c) Is there enough information for you to find the values of $X(15), X(19), X(5)$, and $X(7)$? What are the values that you can decide?

6. The following problems teach the practical spectrum analysis which helps to learn the effects of DFT length, zero padding and windows. (total 3 figures and 6 subplots)

   (a) Take 256 samples of the following signal with a sampling frequency equal to 40 Hz,

$$xa(t) = \cos(8\pi t) + \cos(8.8\pi t) + 0.0001\cos(24\pi t)$$

   First, find period N of the sampled signal $x(n)$. Hand sketch the line spectrum C_n of the signal with the correct frequency label. Now you know exact spectrum of the signal.
   (b) Apply fft() to the first N samples of the signal. It yields the DFS of the signal. Using stem() function to plot the magnitude line spectrum (in dB) with correct frequency axis which should match your hand sketch. (subplot(2,1,1): 1st subplot)
   (c) Apply fft() to the entire 256 samples of the signal and use plot to display the magnitude spectrum in dB scale with correct frequency axis (Hz). (subplot(2,1,2): 2nd subplot) Explain what you see.
   (d) Zero padding the sampled signal $x(n)$ to the length 1024 using fft(x,1024) function. Plot the magnitude spectrum in dB scale with correct frequency axis (Hz). (subplot(2,1,1): 2nd figure 3rd subplot)
   (e) Apply a hanning window, hanning(256), to the signal, $x(n)$, using (.*). Perform fft() to the windowed signal and display the magnitude spectrum in dB scale with correct frequency axis (Hz). (subplot(2,1,2): 2nd figure, 4th subplot)

(f) Zero padding the windowed signal to the length 1024 and plot the magnitude spectrum in dB scale with correct frequency axis (Hz). (subplot(2,1,1): 3rd figure 5th subplot)

(g) Now repeat f) with hamming window, hamming(256). (subplot(2,1,2): 3rd figure 6th subplot).

(h) Now answer the following questions by referring to the plots.

i. Why in 6c), the spectrum of $x(n)$ with 256 samples appears with a lot of ripples compared the spectrum of 6b) with only N (<256) samples?

ii. What is the difference between plots of 6d) and 6c) and why? What is the frequency resolvability (Hz) of the spectrum in 6d) and 6c)?

iii. What is happened to the spectrum when you apply the hanning window to the signal? What is the frequency resolvability (Hz) in this case?

iv. Compare the plots of 6f and 6g)? What are you comments?

7. Prove the orthogonality of the complex exponential:

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nk} \cdot e^{-j\frac{2\pi}{N}nl} = \begin{cases} N & l = k, \mathrm{mod}(N) \\ 0 & l \neq k, \mathrm{mod}(N) \end{cases}$$

(a) Using the orthogonality, to find DFT of $\cos(\frac{l \cdot 2\pi}{N}n)$

8. Consider the linear convolution sum of $y(n) = x(n) * h(n)$ where $x(n) = 0.5^n\,[u(n+1) - u(n-2)]$ and $h(n) = 2n\,[u(n-2) - u(n-4)]$.

(a) Find starting index and ending index of the output sequence $y(n)$.

(b) Calculate the linear convolution sum, using hand draw table. Sketch the data with right time index.

(c) Solve the linear convolution sum using circular convolution and DFT. Compare the result with b).

9. Find the DFT of the following signals:

(a) $x(n) = [\ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ ]$

(b) $y(n) = [\ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ ]$

(c) $z(n) = [\ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ ]$

(d) $w(n) = [\ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ ]$

(e) $q(n) = [\ 0\ \ 0\ \ 0\ \ j\ \ 0\ \ -j\ \ 0\ \ 0\ ]$