EE322 Fourth Homework Assignment

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1. Consider a LTI discrete system with a transfer function $H(z) = 1 - 2z^{-1} + z^{-2}$.

   (a) Find the unit impulse response of the system, $h(n)$.

   (b) Find the difference equation of the system.

   (c) Draw the block diagram of the system.

   (d) Find the frequency response of the system, $H(e^{j\theta}) = \left|H(e^{j\theta})\right| e^{j\angle H(e^{j\theta})}$, and sketch the magnitude and phase frequency response.

   (e) Allocate the poles and zeros of the system in Z-domain with the Unit circle plotted. Explain how do the poles and zeros affect the frequency selectivity.

2. Given a LTI discrete system described by the difference equation, $y(n) = x(n)+x(n-9)$,

   (a) Find the transfer function of the system, $H(z)$.

   (b) Find the poles and zeros of the system.

   (c) Find the frequency response of the system, $H(e^{j\theta}) = \left|H(e^{j\theta})\right| e^{j\angle H(e^{j\theta})}$, and sketch the magnitude and phase frequency response.

   (d) Find the frequency response to the signal $x_a(t) = \cos(270\pi t) + 2 - \sin(90\pi t)$ sampled at $f_s = 810$ (Hz).

3. Given the zeros of a FIR digital filter as follows: $z_0 = -1$, $z_{1,2} = e^{\pm j\frac{\pi}{4}}$.

   (a) Draw the pole/zero diagram.

   (b) Find the difference equation representation of the filter and system transfer function $B(z)$. Can you judge the filter is a linear phase filter or not from its filter coefficient $h(n)$? Is it?

   (c) Sketch the magnitude frequency response $\left|H(e^{j\hat{\omega}})\right|$ with the values clearly marked at $\hat{\omega} = 0, \frac{\pi}{2}$ and π.

   (d) Pick up a proper sampling frequency f_s such that the frequency components 200π and 800π in the following signal will be filtered out by the digital filter.

   $$x_a(t) = 0.2\sin(200\pi t + 1) - 0.3\cos(800\pi t - 1.5) + 2\cos(70\pi t - 1)$$

   and find the system steady state response $y_{ss}(n)$ to the signal.

   (e) Find the expression of the phase frequency response $\angle H(e^{j\hat{\omega}})$ and group delay $\tau(\hat{\omega})$.

(f) Verify the results using Matlab^{TM}. You may use functions such as filter(), freqz(), poly(), fft(), abs(), angle(), impz() and zplane(), etc.

4. In the following signal

$$x_a(t) = e^{-0.5t} + 0.5\cos(400\pi t - 1) + 0.03\cos(1200\pi t - \frac{\pi}{2})$$

where $0 \le t \le 2$, the sinusoids are considered interferences to a low–pass type signal $e^{-0.5t}$. Design a Notch (Nulling) filter $B(z) = 1 + z^{-L}$ and a proper sampling frequency to eliminate the sinusoidal interferences.

(a) Show the pole/zero diagram using zplane() function.

(b) Plot frequency response (both magnitude and phase) of the filter in linear scale with frequency label in Hz. Plot the magnitude of the Fourier transform of $x(n)$ with frequency axis labeled in Hz, that is, k=0:N–1; f=k*fs/N; where N is the length of $x(n)$.

(c) Using subplot() to show the plots of the original signal $x_a(t)$, filtered signal $y(n)$, and desired signal $e^{-0.5t}$.

(d) Plot the magnitude of the Fourier transform of the filtered signal $y(n)$ with frequency axis labeled in Hz.

(e) Draw conclusions from the observation.

5. Show that the following moving average filter, $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$ is a linear phase FIR filter (find $\angle H(e^{j\theta})$).

6. Given an IIR filter, $y(n) = 1.7y(n-1) - 0.72y(n-2) + x(n) + x(n-1)$:

(a) Find the transfer function, $H(z) = \frac{B(z)}{A(z)}$.

(b) Draw the Direct I realization block diagram.

(c) Find and draw pole/zero diagram of the system in Z-domain with the Unit Circle identified.

(d) Sketch the magnitude frequency response with normalized frequency label, θ and real frequency label with $f_s = 1$ KHz.

(e) Assume the input signal, $x(n) = u(n)$, calculate the first 6 samples of the output sequence with the initial conditions: $y(-1) = 1$ and $y(-2) = -1$.

(f) Using Matlab function [H,faxis]=freqz(B,A,256,fs) to verify your result in (6d).

7. Determine $H(z)$ using the impulse invariant principle given that

$$H_a(s) = \frac{1}{s^2 + 5s + 6}.$$

8. Determine $H(z)$ using the impulse invariant principle given that

$$H_a(s) = \frac{s}{(s+2)(s+1)^2}.$$

9. Determine $H(z)$ using the impulse invariant principle given that

$$H_a(s) = \frac{3}{s^2(s+2)}.$$

10. An IIR filter has the following transfer function

$$H(z) = \frac{1-1.1z^{-1}}{1-0.3z^{-1}-0.4z^{-2}}.$$

   (a) Plot the pole–zero diagram.
   (b) Find all the possible R.O.C. for $H(z)$ and indicate all the R.O.C. on the pole–zero diagram.
   (c) Which one of the R.O.C. corresponding to a stable filter? Why?
   (d) Which one of the R.O.C. corresponding to a causal filter? Why?
   (e) Find the impulse response, $h(n)$, of the stable filter using inverse Z–transform
   (f) Use Matlab^{TM} function impz() to generate first 10 values of the impulse response and compare them with $h(n)$ that you find using inverse Z transform.
   (g) Sketch the magnitude frequency response of the filter according to its pole–zero diagram. What kind of frequency selective filter is this?
   (h) Calculate the steady state response to a sinusoidal signal which has a frequency equal to 3/8 of the sampling frequency.
   (i) For an input signal $x(n) = 2(0.5)^n u(n)$ and initial conditions $y(-1) = 1$, $y(0) = -0.5$, calculate the first 10 values of $y(n)$ iteratively by hand.
   (j) Now for the same input signal and same initial conditions, calculate the first 10 values of $y(n)$ by using Matlab^{TM} functions filtic() and filter().