EE322 First MidTerm Exam

Spring 2000

Instructor: Kefu Xue, Ph.D.

Instructions: You are permitted to use a **self-prepared** study-guide limited to one ($8\frac{1}{2}$" $\times$ 11") page (both sides). The duration of this test is 75 minutes. Show all the intermediate steps for credits.

1. (30 pts.) Given a signal composed of sinusoids:

$$x_a(t) = 2\sin(240\pi t + \frac{\pi}{3}) + \sin(320\pi t - \frac{\pi}{4}) - \cos(560\pi t + 1)$$

   (a) Is $x_a(t)$ periodic? If so, what is the period? Hint: Find the fundamental frequency ω_0.

   (b) Find the Fourier series coefficients using Fourier series recognition method (Euler's relation). Hint: Total of 6 coefficients.

   (c) Sketch the two–sided spectrum representation of the signal $x_a(t)$.

   (d) Find the expression of the sampled sequence $x(n)$ with a sampling frequency that is twice of the signal's Nyquist frequency.

   (e) Is the sequence $x(n)$ periodic? If yes, find the minimum period (in number of samples) of the sequence. If not, explain why.

   (f) How many periods of $x_a(t)$ are covered in one period of samples $x(n)$?

2. (10 pts.) For a given chirp signal $c_a(t) = \cos(20\pi t^2 + \beta t + \phi)$, $0 \le t \le T_2$, find coefficient values β, ϕ and T_2 such that the signal, $c_a(t)$, starts with a zero value, i.e., $c_a(0) = 0$ and the frequency sweeps from 50 Hz to 250 Hz during the time.

3. (20 pts.) Given the discrete Fourier series representation, $X(k)$, of a signal,

$$X(0) = 512, \quad X(2) = X^*(254) = 128e^{j\frac{\pi}{2}}, \quad X(5) = X^*(251) = 256e^{j\frac{\pi}{2}},$$

   where $k = 0, 1, \cdots, 255$,

   (a) find the signal $x(n)$. Hint: $e^{j\frac{l\cdot 2\pi}{N}n} \Leftrightarrow N\delta(k-l)$.

   (b) Is $x(n)$ periodic and if yes, what is the period N?

   (c) If a sampling frequency 1024 Hz is selected, sketch the two sided spectrum representation of the signal $x_a(t)$. Hint: $\hat{\omega} = \omega T$

   (d) What is the reconstructed signal $x_a(t)$?

4. (10 pts.)Given a signal $x_a(t) = 8\cos(48\pi t - 2)$ and sampling frequency $f_s = 100$ Hz,

(a) find expression of $x(n)$ and its period N.

(b) Find two other analog signals $y_a(t)$ and $w_a(t)$ such that $x(n) = y(n) = w(n)$ if the same $f_s = 100$ Hz is used. Explain your selections.

5. (10 pts.) Simplify $x_a(t) = 5\cos(20\pi t - 2) + \sin(20\pi t + 1) - 3\cos(20\pi t - 0.5)$ to the standard form $A\cos(\omega_0 t + \phi)$.

6. (20 pts.) Sample $x_a(t) = \cos(200\pi t)\cos(1000\pi t)$ with 5 samples in each period.

   (a) What is the fundamental frequency, ω_0, of the signal? Hint: $2\cos(\omega_1 t)\cos(\omega_2 t) = \cos[(\omega_1 + \omega_2)t] + \cos[(\omega_1 - \omega_2)t]$

   (b) What is the sampling frequency?

   (c) Is there aliasing error? Why? If there is aliasing error, explain to what frequency the highest frequency component is folded.

   (d) What will be reconstructed signal?

   (e) What is your suggested number of samples in each period to avoid aliasing error? Explain your selection.

7. **Optional Take-home Matlab^{TM} Parts for Extra 10 Bonus Points:** Following the problem 6,

   (a) Plot signal $x_a(t)$ with the sampling frequency corresponding to 5 samples in each period. (10 periods)

   (b) Plot the magnitude spectrum of the above signal. (labeled with frequency (Hz))

   (c) Plot signal $x_a(t)$ with a newly selected sampling frequency without aliasing error. (10 periods)

   (d) Plot the magnitude spectrum of the above signal. (labeled with frequency (Hz))

Put the 4 plots in one figure using subplot(4,1,n) and each plot should have a proper title(). If you chose to complete the optional problem, you need to solve this problem independently and email me (kxue@cs.wright.edu) your m–file no later than 4 pm Wed. 4/19/2000. I won't debug your program. If your program does not run or figure does not show the correct results, you will not get the bonus points.

Please hand in your one-page study guide with the test book to avoid 20% off penalty.