EE322 Second MidTerm

(1) (a) $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$

$= 1 \cdot z + 2 - 1 \cdot z^{-1} + 3z^{-3}$

$= z + 2 - z^{-1} + 3z^{-3}$ R.O.C. $0 < |z| < \infty$

(b) $H(z) = 2z^{-2} + 1 - 0.5z^{-4}$

$h(n) = 2 \quad 1 \quad -0.5$

$(n=2) \quad (n=0) \quad (n=4)$

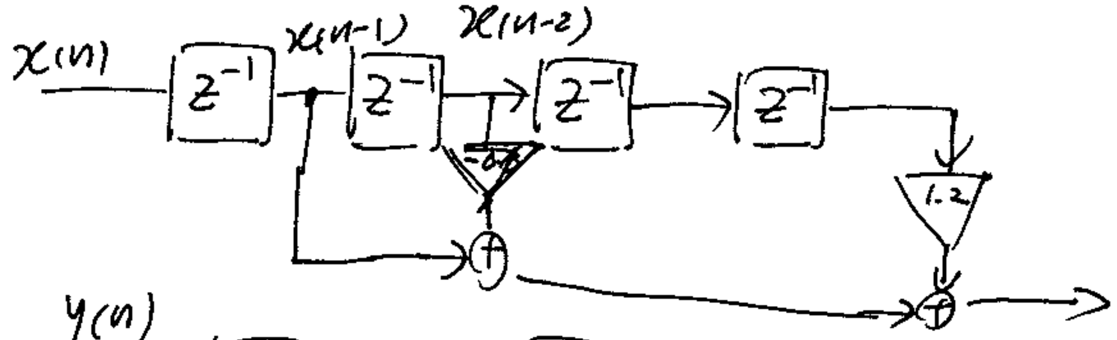
$= [\,1 \quad 0 \quad 2 \quad 0 \quad -0.5\,]$

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4$

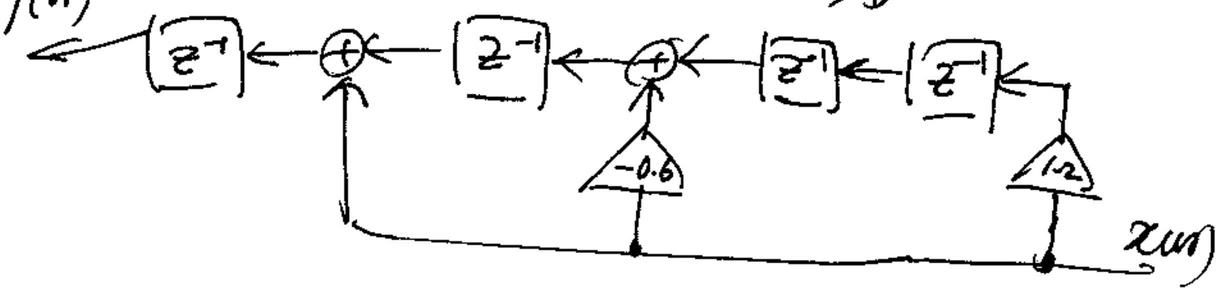
(2) $H(z) = B(z) = z^{-1} - 0.6z^{-2} + 1.2z^{-4}$

(a) $y(n) = x(n-1) - 0.6\,x(n-2) + 1.2\,x(n-4)$

(b)



(c)



3) $h(n) = 2^n \; ; \; n = 1, 2, 3, 4$

$= [\,2 \quad 4 \quad 8 \quad 16\,]$

$n = \; 1 \quad 2 \quad 3 \quad 4$

$x(n) = [\,1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1\,]$

$n = \; -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

(a) $S_y = -2 + 1 = -1$

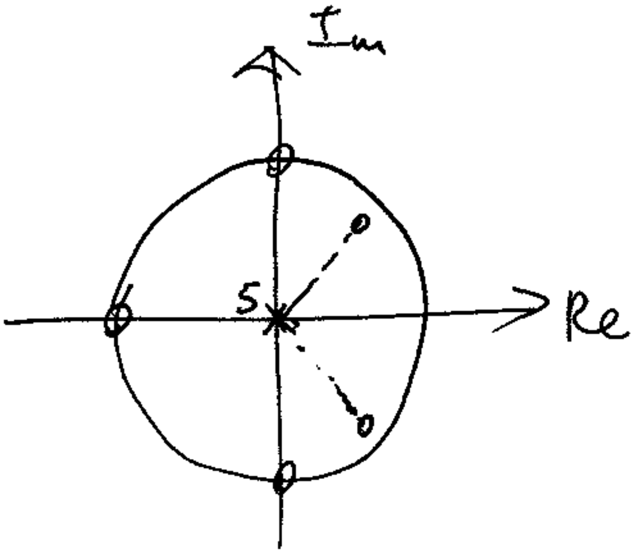
$E_y = 3 + 4 = 7$

$L_y = L_x + L_h - 1 = 9$

			1	-1	1	-1	1	-1		
16	8	4	2							2
	16	8	4	2						2
		16	8	4	2					6
			16	8	4	2				10
					⋮					-10
										10
										-12
										8
										-16

$y(n) =$	[2	2	6	10	-10	10	-12	8	-16]
$n =$	-1	0	1	2	3	4	5	6	7

(4) (a)



(b)

$$H(z) = (1 - z_1 z^{-1})(1 - z_2 z^{-1})(1 - z_3 z^{-1})(1 - z_4 z^{-1})(1 - z_5 z^{-1})$$

$$= (1 - 2\,\mathrm{Re}\{z_1\} z^{-1} + |z_1|^2 z^{-2})(1 + |z_3|^2 z^{-2})(1 + z^{-1})$$

$$= (1 - 2 \cdot 0.9 \cos(\tfrac{\pi}{4}) z^{-1} + 0.81 z^{-2})(1 + z^{-2})(1 + z^{-1})$$

$$= (1 - 0.9 \times \sqrt{2}\, z^{-1} + 0.81 z^{-2})(1 + z^{-1} + z^{-2} + z^{-3})$$

$$= (1 - 0.9\sqrt{2}\, z^{-1} + 0.81 z^{-2}$$

$$z^{-1} - 0.9\sqrt{2}\, z^{-2} + 0.81 z^{-3}$$

$$z^{-2} - 0.9\sqrt{2}\, z^{-3} + 0.81 z^{-4}$$

$$z^{-3} - 0.9\sqrt{2}\, z^{-4} + 0.81 z^{-5}$$

$$= 1 - 0.2728 z^{-1} + 0.5372 z^{-2} + 0.5372 z^{-3} - 0.4628 z^{-4} + 0.81 z^{-5}$$

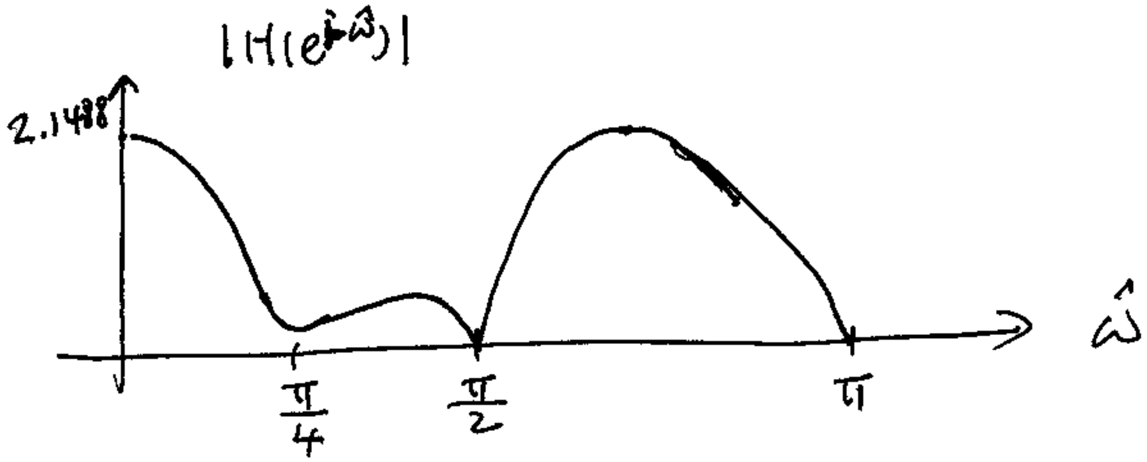
$$b = [1 \quad -0.2728 \quad 0.5372 \quad 0.5372 \quad -0.4628 \quad 0.81]$$

(c)

$$H(1) = 1 - 0.2728 + 0.5372 + 0.5372 - 0.4628 + 0.81$$

$$= 2.1488$$

$$H(j) = 0, \quad H(-1) = 0$$



d) It is a Bandstop filter.

(e) $x(n) = 2 + 2\sin\left(\frac{200\pi}{800}n + \frac{\pi}{4}\right) - 3\cos\left(\frac{400\pi}{800}n + 2\right)$

$= 2 + 2\sin\left(\frac{\pi}{4}n + \frac{\pi}{4}\right) - 3\cos\left(\frac{\pi}{2}n + 2\right)$

$y_{ss}(n) = 2 \times 2.1488 + 2M\cos\left(\frac{\pi}{4}n - \frac{\pi}{2} + \frac{\pi}{4} + \varphi\right)$

$M = |H(e^{j\frac{\pi}{4}})|, \quad \varphi = \angle H(e^{j\frac{\pi}{4}})$

$H(e^{j\frac{\pi}{4}}) = 1 - 0.2728e^{-j\frac{\pi}{4}} + 0.5372e^{-j\frac{\pi}{2}}$
$+ 0.5372e^{-j\frac{3\pi}{4}} - 0.4628e^{-j\pi}$
$+ 0.81e^{-j\frac{5\pi}{4}}$

$= 1 - 0.2728e^{-j\frac{\pi}{4}} - 0.5372j + 0.5372e^{-j\frac{3\pi}{4}}$
$+ 0.4628 + 0.81e^{j\frac{3\pi}{4}} = 0.3205 - j0.1437$

$= 0.3512e^{-j0.4216}$

$y_{ss}(n) = 4.2976 + 0.7024\cos\left(\frac{\pi}{4}n - \frac{\pi}{4} - 0.4216\right)$

(5)

$$H(z) = \frac{1}{N} \sum_{k=0}^{N-1} z^{-k}$$

$$= \frac{1}{10} \sum_{k=0}^{9} z^{-k}$$

$$h(n) = \frac{1}{10} [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$$

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$

$\therefore \quad = h(10-1-n)$ even symmetry

(a) $\therefore$ $H(z)$ is a linear phase filter

(b) Linear phase : $\angle H(e^{j\hat{\omega}}) = -\left(\frac{N-1}{2}\right)\hat{\omega} + \beta$

Group delay : $\tau(\hat{\omega}) = -\frac{d \angle H(e^{j\hat{\omega}})}{d\hat{\omega}} = \frac{N-1}{2} = \frac{10-1}{2}$

$= 4.5$ (samples)

(c)

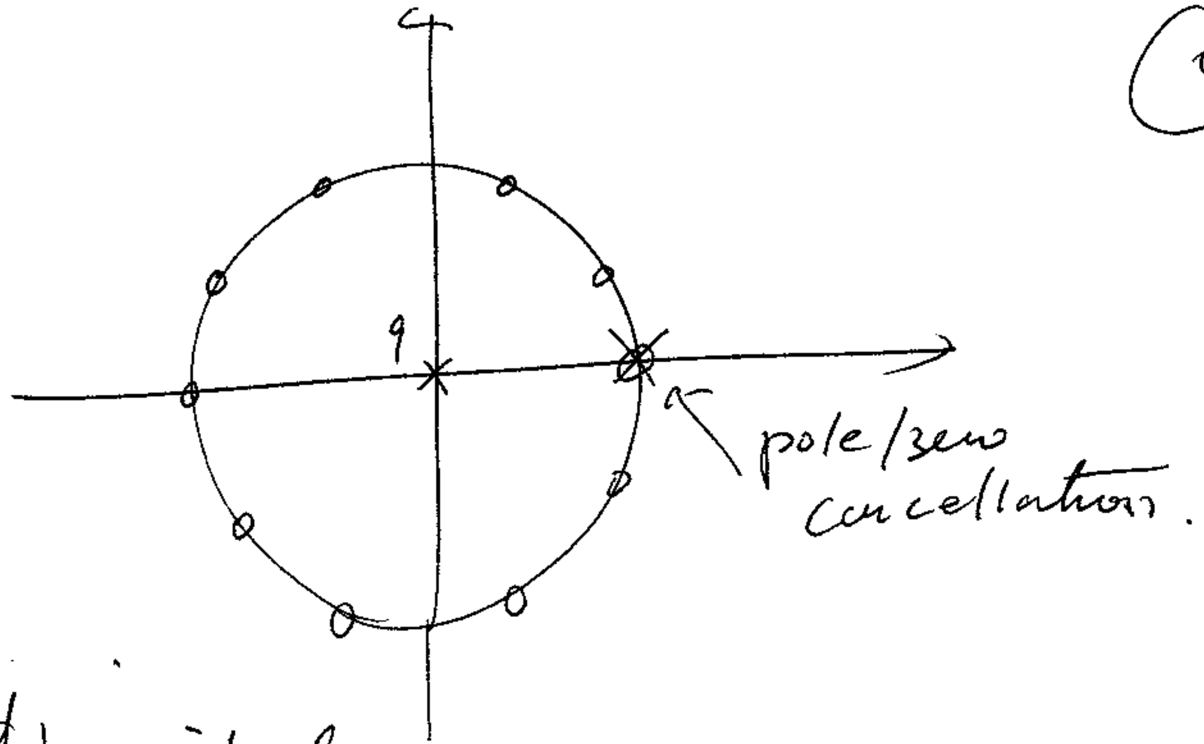
$$H(z) = \frac{1}{10} \sum_{k=0}^{9} z^{-k} = 0$$

$$= \frac{1}{10} \frac{1 - z^{-10}}{1 - z^{-1}}$$

zeros: $\quad 1 - z^{-10} = 0 \quad z^{-10} = 1 = e^{j2\pi k}$

$$z_k = e^{-j\frac{2\pi}{10}k} \quad k = 0, 1, \cdots, 9$$

poles: $\quad p_k = 1$



d) if $f_s = 10\,\text{KHz}$,

$$\hat{\omega}_k = \frac{2\pi}{10}\cdot k = \omega_k \cdot T = \frac{\omega_k}{f_s}$$

Nulls frequency $\omega_k = \frac{2\pi}{10}\cdot k \times 10\,\text{KHz}$ $\quad k = 1, 2, \cdots 9$

$= 2\pi \cdot k$ (Kradian/second)

or k (kHz)

1 KHz, 2 KHz, 3 KHz --- 9 KHz

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$f_s > 2 \times 360\,\text{Hz}$ $\qquad H(z) = 1 - z^{-L}$

$$\frac{2\pi \cdot 60}{f_s} = \frac{2\pi}{L} \qquad L \cdot 60 = f_s$$

$L\,60 > 720\,\text{Hz}$ $\qquad L > 12$. Let $L = 13$

$f_s = 420\,\text{Hz}$ $\qquad H(z) = 1 - z^{-13}$