EE 322/522 Chapt. 8 & 9 Test Samples

FALL 2000

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Instructions: Duration: $D = 120$ minutes. You are permitted to use a self-prepared study-guide limited to two (8 1/2" $\times$ 11") pages (both sides). Please include intermediate procedures for credits.

This test is printed on both sides, please turn the paper over for more points. Total 8 problems and 100 pts. and plus one 10 pts. optional bonus problem. When you finish the test, please turn in this test paper with your test book, otherwise you will receive 0 points for your test score. You can view (but can not keep) your test result at Room 311 Russ Center in the middle of next week.

1. (10 pts.) A digital filter has two poles and one zero: $z_1 = -1$, $p_{1,2} = 0.9e^{\pm j\frac{3\pi}{4}}$. Find the transfer function $H(z)$ of the filter and sketch the magnitude frequency response of the filter.

2. (10 pts.)Given a sequence
$$x(n) = [1 \quad -2 \quad -1 \quad 3]$$
that is sampled from an analog signal using a sampling frequency of 2 KHz,
   (a) Find the values of DFT $X(k)$;
   (b) Find the frequency resolution of the DFT spectrum;
   (c) Find the frequencies that each sample of the DFT represents.

3. (15 pts.) DFT short problems:
   (a) Given an analog signal with a bandwidth of 1500Hz and a signal duration of 200ms, find a sampling frequency and its corresponding number of samples and frequency resolution in DFT analysis.
   (b) 256 samples of an analog signal is collected with an sampling interval of 0.1 ms. What is the signal duration? What is the corresponding frequency resolution in DFT? How many zeros should be padded, if a desired DFT frequency resolution should be finer than or equal to 2 Hz?
   (c) A GPS (global positioning system) signal is sampled at 6 MHz for 3 ms. How many samples are collected? What is the highest frequency that presents in the Fourier spectrum (in Hz)? To what frequencies, do DFT samples $X(9)$, $X(154)$, and $X(1782)$ represent respectively?

4. (10 pts.)Part of a 4 points DFT $\{X(k), \quad k = 0, 1, 2, 3\}$ of a real sequence $x(n)$ is
$$X(0) = 2; \quad X(2) = 0; \quad X(3) = 1$$
   (a) If $X(2)$ represent the frequency component of 50 Hz, what is the sampling frequency that is used in sampling signal $x_a(t)$? Explain your decision.
   (b) Find the values of $\{x(n), \quad n = 0, 1, 2, 3\}$. (Hint: find DFT component $X(1)$ first.)

5. Z-transforms

   (a) (10 pts.) Find the impulse response $h(n)$ from the following stable system transfer function:

$$H(z) = \frac{1 + 0.5z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

   Note: credit will be given only if a general term (closed form) of sequence $h(n)$ is solved. (Hint: find the poles (all real numbers) and the R.O.C. from the given information.)

6. (15 pts.) An ECG (electrocardiogram) signal with bandwidth equal to 200 Hz has been interfered by the 60 Hz power line frequency. Select a sampling frequency and design a linear phase FIR comb filter to remove the 60 Hz frequency and up to its 5th harmonics within the folding frequency. Note: selecting a sampling frequency is part of the design of a comb filter.

7. (10 pts.) Determine $H(z)$ using the impulse invariant principle given that

$$H_a(s) = \frac{2}{s^2(s+2)}$$

8. (10 pts. Bonus!) Given the discrete Fourier series representation, $X(k)$, of a discrete–time signal, $x(n)$

$$X(0) = 512, \quad X(2) = X^*(126) = 128e^{j\frac{\pi}{2}}, \quad X(5) = X^*(123) = 256e^{j\frac{\pi}{2}},$$

   where $k = 0, 1, \cdots, 127$,

   (a) find the discrete–time signal $x(n)$. Hint: $e^{j\frac{l\cdot 2\pi}{N}n} \Leftrightarrow N\delta(k-l)$ and $\cos(\frac{l\cdot 2\pi}{N}n) \Leftrightarrow \frac{N}{2}[\delta(k-l) + \delta(k+l)]$. Is $x(n)$ periodic and if yes, what is the period N?

   (b) If the signal was sampled by a sampling frequency of 1 KHz, sketch the two sided spectrum representation of the signal $x_a(t)$. Hint: $\hat{\omega} = \omega T$. What is the original signal $x_a(t)$?