PROB. 11-16

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$a = K\left[X - \left(\frac{A}{X}\right)\right] \frac{ft}{s^2}, \quad v_0 = 0 \ @ \ X_0 = 1^{ft}$

$v = v_1 \ @ \ X = 2^{ft}, \quad v = 2v_1 \ @ \ X = 8^{ft}$

$v = 29 \frac{ft}{s} \ @ \ X = 16^{ft}$, FIND A AND K

$K \cdot ft = \frac{ft}{s^2}, \ [K] = \frac{1}{s^2} \ ; \ \frac{A}{ft} = ft, \ A = ft^2$

$a = f(x)$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_{X_0}^{X} f(x) = \int_{X_0}^{X} K\left[X - \left(\frac{A}{X}\right)\right] dX$$

$$= K\left[\frac{X^2}{2} - A \ln X\right]_{X_0}^{X}$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = K\left[\frac{1}{2}(X^2 - X_0^2) - A \ln\left(\frac{X}{X_0}\right)\right]$$

$$\frac{1}{2}v^2 - \frac{1}{2}(0)^2 = K\left[\frac{1}{2}(X^2 - 1^2) - A \ln\left(\frac{X}{1}\right)\right] \quad v = 0 \ @ \ X = 1$$

$$\frac{1}{2}v^2 = K\left[\frac{1}{2}(X^2 - 1) - A \ln X\right]$$

$$\frac{1}{2}v_1^2 = K\left[\frac{1}{2}(2^2 - 1) - A \ln 2\right] \quad v = v_1 \ @ \ X = 2$$

$$v_1^2 = 2K\left(\frac{3}{2} - A \ln 2\right)$$

$$\frac{1}{2}(2v_1)^2 = K\left[\frac{1}{2}(8^2 - 1) - A \ln 8\right] \quad v = 2v_1 \ @ \ X = 8$$

$$2v_1^2 = K\left(\frac{63}{2} - A \ln 8\right)$$

PROB. 11-16 CONT.

$$v_1^2 = \frac{K}{2}\left(\frac{63}{2} - A \ln 8\right)$$

$$2\cancel{K}\left(\frac{3}{2} - A \ln 2\right) = \frac{\cancel{K}}{2}\left(\frac{63}{2} - A \ln 8\right)$$

$$3 - 2A \ln 2 = \frac{63}{4} - \frac{1}{2} A \ln 8$$

$$\frac{1}{2} A \ln 8 - 2A \ln 2 = \frac{63}{4} - 3$$

$$A\left(\frac{1}{2} \ln 8 - 2 \ln 2\right) = \frac{63}{4} - 3$$

$$A = \frac{\frac{63}{4} - 3}{\frac{1}{2} \ln 8 - 2 \ln 2} \boxed{= -36.79 \text{ ft}^2}$$

$$\frac{1}{2} v^2 = K\left[\frac{1}{2}(x^2 - 1) - A \ln x\right]$$

$$K = \frac{\frac{1}{2} v^2}{\frac{1}{2}(x^2 - 1) - A \ln x}$$

$$K = \frac{\frac{1}{2}(29)^2}{\frac{1}{2}(16^2 - 1) - (-36.79) \ln 16} \boxed{= 1.832 \frac{1}{s^2}}$$

$v = 29$ @ $x = 16$