

PROB. 11-25

$$a = 0.4(1 - kv), \quad v_0 = 0 \text{ @ } t_0 = 0 \text{ s and } x_0 = 4 \text{ m,}$$

$$v = 4 \frac{\text{m}}{\text{s}} \text{ @ } t = 15 \text{ s, FIND } k, \text{ X WHEN } v = 6 \frac{\text{m}}{\text{s}}, v_{\text{max}}$$

$$a = f(v), \quad a = \frac{dv}{dt} = 0.4(1 - kv)$$

$$\int_0^t dt = \int_0^v \frac{dv}{0.4(1 - kv)}$$

$$t = \frac{1}{0.4} \int_0^v \frac{dv}{(1 - kv)}$$

$$\text{LET } w = 1 - kv, \quad dw = -kdv, \quad dv = \left(-\frac{1}{k}\right)dw$$

$$\text{LIMITS: @ } v = 0, w = 1; \quad \text{@ } v = v, w = 1 - kv$$

$$t = \frac{1}{0.4} \int_1^{1 - kv} \left(-\frac{1}{k}\right) \frac{dw}{w}$$

$$t = -\frac{1}{0.4k} \ln(1 - kv)$$

$$-0.4kt = \ln(1 - kv)$$

$$v = 4 \text{ @ } t = 15:$$

$$-0.4k(15) = \ln(1 - k \cdot 4)$$

$$6k + \ln(1 - 4k) = 0$$

TRANSCENDENTAL EQUATION FOR K

$$k \approx 0.1455 \frac{\text{s}}{\text{m}}$$

K	f(K)
0.1	0.08917
0.2	-0.4097
0.11	0.08018
0.13	0.04603
0.15	-0.01629
0.14	0.01902
0.145	0.002499
0.146	-0.001070
0.1455	0.0007261

PROB. 11-25 CONT.

FIND x WHEN $v = 6 \frac{m}{s}$

$$dx = \frac{v dv}{f(v)}$$

$$\int_{x_0}^x dx = \int_{v_0}^v \frac{v dv}{0.4(1-Kv)}$$

$$x - x_0 = \frac{1}{0.4} \int_{v_0}^v \frac{v dv}{(1-Kv)}$$

$$\int \frac{x dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2} \ln(ax+b)$$

$$a = -K, \quad b = 1$$

$$x - x_0 = \frac{1}{0.4} \left[\frac{v}{(-K)} - \frac{(1)}{(-K)^2} \cdot \ln(1-Kv) \right]_{v_0}^v$$

$$x - x_0 = \frac{1}{0.4} \left[-\frac{1}{K}(v - v_0) - \frac{1}{K^2} \cdot \ln\left(\frac{1-Kv}{1-Kv_0}\right) \right]$$

$$x = (4^m) + \frac{1}{0.4} \left[-\frac{1}{0.1455}(6-0) - \frac{1}{(0.1455)^2} \cdot \ln\left(\frac{1-0.1455 \cdot 6}{1-0.1455 \cdot 0}\right) \right]$$

$$x = 144.6 \frac{m}{s}$$

v_{\max} OCCURS WHEN $\frac{dv}{dt} = 0$

$$a = \frac{dv}{dt} = 0 = 0.4(1-Kv_{\max})$$

$$v_{\max} = \frac{1}{K} = \frac{1}{0.1455 \frac{2}{m}} = 6.873 \frac{m}{s}$$