PROB. 12-76

$r = r_0/\sqrt{\cos 2\theta} = r_0(\cos 2\theta)^{-\frac{1}{2}}$

USING EQN. (12.27): $r^2\dot{\theta} = h$, FIND v_r, v_θ

$v = v_0$ @ $r = r_0$

FOR A PARTICLE WITH A CENTRAL FORCE;

$r^2\dot{\theta} = h = \text{CONSTANT}$

$v_\theta = r\dot{\theta}$

$r \cdot v_\theta = \text{CONSTANT} = r_0 v_0$

$$v_\theta = \frac{r_0 v_0}{r} = \frac{r_0 v_0}{r_0(\cos 2\theta)^{-\frac{1}{2}}} \boxed{= v_0\sqrt{\cos 2\theta}}$$

$v_r = \frac{dr}{dt}$: SINCE $r = r(\theta)$, USE THE CHAIN RULE:

$$v_r = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dr}{d\theta}\cdot\dot{\theta}$$

$$\frac{dr}{d\theta} = -\frac{1}{2}r_0(\cos 2\theta)^{-\frac{3}{2}}\cdot\left[2(-\sin 2\theta)\right] = \frac{r_0 \sin 2\theta}{(\cos 2\theta)^{\frac{3}{2}}}$$

$$\dot{\theta} = \frac{v_\theta}{r} = \frac{v_0\sqrt{\cos 2\theta}}{r_0(\cos 2\theta)^{-\frac{1}{2}}} = \frac{v_0}{r_0}\cdot\cos 2\theta$$

$$v_r = \left[\frac{r_0 \sin 2\theta}{(\cos 2\theta)^{\frac{3}{2}}}\right]\cdot\left[\frac{v_0 \cos 2\theta}{r_0}\right] \boxed{= v_0\cdot\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}}$$