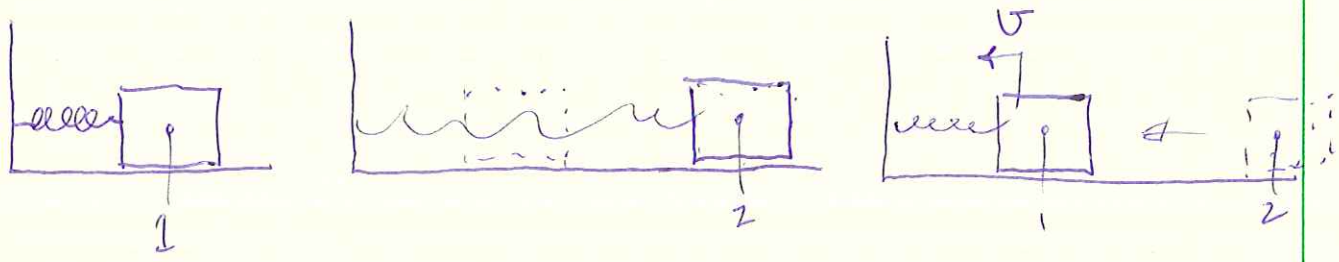
PROB. 13-26

$W = 10^{LB}$, $K = \left(12 \frac{LB}{IN}\right)\left(\frac{12\,IN}{ft}\right) = 144 \frac{LB}{ft}$, $\mu_K = 0.4$

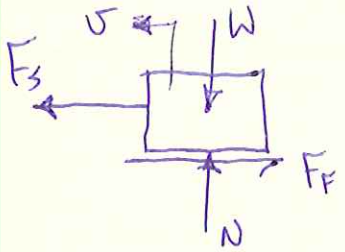
a) FIND SPEED AT UNSTRETCHED POSITION:



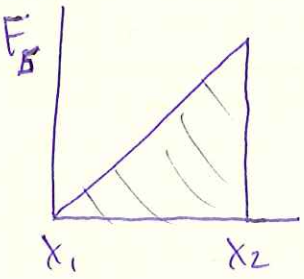
AT X_2, SPRING FORCE: $F = K\Delta X$

$\Delta X = \frac{F}{K} = \frac{(20^{LB})}{\left(144 \frac{LB}{IN}\right)} = 0.1389\ ft$

FBD POSITION 2: RELEASE BLOCK



SPRING FORCE:



$_2U_1 = T_1 - T_2$, $T_2 = 0$ SINCE $v_2 = 0$

$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}\left(\frac{W}{g}\right) v_1^2$

$_2U_1 = F\cos\alpha \cdot \Delta X = (F_S - F_F)\Delta X = \int_2^1 F_S\,dx - F_F \cdot \Delta X$

SPRING WORK: $(_2U_1)_S = \frac{1}{2} F_2 \cdot \Delta X$ AREA UNDER CURVE

FRICTION WORK: $(_2U_1)_F = -F_F \cdot \Delta X = -\mu_K W \cdot \Delta X$

$_2U_1 = \frac{1}{2} F_2 \cdot \Delta X - \mu_K W \cdot \Delta X = \left(\frac{1}{2} F_2 - \mu_K W\right)\Delta X$

PROB. 13-26 CONT.

$$\left(\frac{1}{2}F_2\,\Delta x - \mu_k W\right)\Delta x = \frac{1}{2}\left(\frac{W}{g}\right)v_1^2$$

$$v_1 = \sqrt{\frac{g\Delta x}{W}\left(F_2 - 2\mu_k W\right)}$$

$$v_1 = \sqrt{\frac{\left(32.2\ \frac{ft}{s^2}\right)\left(0.1389\ ft\right)}{\left(10^{LB}\right)}\cdot\left[\left(20^{LB}\right) - 2\left(0.4\right)\left(10^{LB}\right)\right]}$$

$$\boxed{v_1 = 2.317\ \frac{ft}{s}}$$

b) FIND MAXIMUM SPEED: $_2U_x = T_x - T_2$

$T_2 = 0,\quad T_x = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{W}{g}\right)v^2$

$$_2U_x = F\cos\alpha\cdot\Delta x = \left(F_S - F_F\right)\cdot\Delta x = \int_2^x F_S\,dx - F_F\,\Delta x$$

SPRING WORK: $\left(_2U_x\right)_S = \frac{1}{2}\left(F_x + F_2\right)\left(x_2 - x\right)$ AREA UNDER CURVE

$$\left(_2U_x\right)_S = \frac{1}{2}\left(kx + kx_2\right)\left(x_2 - x\right) = \frac{1}{2}k\left(x_2^2 - x^2\right)$$

$$\left(_2U_x\right)_F = -\mu_k W\left(x_2 - x\right)$$

$$\frac{1}{2}k\left(x_2^2 - x^2\right) - \mu_k W\left(x_2 - x\right) = \frac{1}{2}\left(\frac{W}{g}\right)v^2$$

$$v^2 = \left(\frac{g}{W}\right)k\left(x_2^2 - x^2\right) - 2\mu_k g\left(x_2 - x\right)$$

v_{MAX} OCCURS AT $dv/dx = 0$: TAKE DERIVATE W.R.T. x:

$$2v\frac{dv}{dx} = -\frac{2gk}{W}\cdot x + 2\mu_k g$$

SET $dv/dx = 0$

PROB. 13-26 CONT.

$$-\frac{2gK}{W}\cdot X + 2\mu_K g = 0$$

$$X = \frac{\mu_K W}{K} = \frac{(0.4)(10^{LB})}{\left(144 \frac{LB}{FT}\right)} = 0.02778\ ft$$

$$v_{max} = \sqrt{\frac{\left(32.2 \frac{ft}{s^2}\right)\left(144 \frac{LB}{ft}\right)}{(10^{LB})} \cdot \left[(0.1389\ ft)^2 - (0.02778\ ft)^2\right] - 2(0.4)\left(32.2 \frac{ft}{s^2}\right)\left[(0.1389\ ft) - (0.02778\ ft)\right]}$$

$$\boxed{v_{max} = 2.393\ ft}$$