PROB. 13-85

$R = 6370\ km = 6.37 \times 10^6\ m$

$r_1 = 6.37 \times 10^6 + (300\ km)\left(\frac{1000\ m}{km}\right) = 6.67 \times 10^6\ m$

$m = 3600\ kg$

a) FIND ADDITIONAL ENERGY TO RAISE SATELLITE TO $r_2 = 6.37 \times 10^6 + (35{,}770\ km)\left(\frac{1000\ m}{km}\right) = 4.214 \times 10^7\ m$

ENERGY AT POINT 1:

$E_1 = T_1 + V_1 = \frac{1}{2} m v_1^2 - \frac{GMm}{r} = \frac{1}{2} m v_1^2 - \frac{gR^2 m}{r_1}$

$\Sigma F_n = m \frac{v_n^2}{\rho}, \quad \frac{GMm}{r_1^2} = m \frac{v_1^2}{r_1}, \quad v_1^2 = \frac{GM}{r_1} = \frac{gR^2}{r_1}$

$E_1 = \frac{1}{2} m \left(\frac{gR^2}{r_1}\right) - \frac{gR^2 m}{r_1} = -\frac{gR^2 m}{2 r_1}$

$E_1 = -\frac{\left(9.81\ \frac{m}{s^2}\right)(6.37 \times 10^6\ m)^2 (3600\ kg)}{2(6.67 \times 10^6\ m)} = -1.074 \times 10^{11}\ J$

$E_2 = -\frac{gR^2 m}{2 r_2}$

$E_2 = -\frac{\left(9.81\ \frac{m}{s^2}\right)(6.37 \times 10^6\ m)^2 (3600\ kg)}{2(4.214 \times 10^7\ m)} = -1.700 \times 10^{10}\ J$

$\Delta E = E_2 - E_1 = (-1.7 \times 10^{10}) - (-1.074 \times 10^{11})$

$\Delta E = (9.040 \times 10^{10}\ J)\left(\frac{GJ}{10^9\ J}\right) = \boxed{90.4\ GJ}$

b) FIND ADDITIONAL ENERGY FROM LAUNCH TO G.E.O.

PROB. 13-85 CONT.

AT EARTH'S SURFACE,

$E_3 = T_3 + V_3$

$V_3 = 0 \therefore T_3 = 0$

$$E_3 = -\frac{GMm}{r} = -\frac{gR^2m}{R} = -gRm$$

$$E_3 = -\left(9.81 \frac{m}{s^2}\right)(6.37 \times 10^6 \text{ m})(3600 \text{ kg}) = -2.250 \times 10^{11} \text{ J}$$

$$\Delta E = E_2 - E_3 = (-1.7 \times 10^{10}) - (-2.25 \times 10^{11})$$

$$\Delta E = (2.080 \times 10^{11} \text{ J})\left(\frac{6 \text{ J}}{10^9 \text{ J}}\right) = \boxed{208 \text{ GJ}}$$