PROB. 13-95

$W = 10^{LB}$, $K = 50\,\frac{LB}{ft}$, $l_u = \frac{3}{2}\,ft$, $r_A = 1^{ft}$,

$(v_\theta)_A = 16\,\frac{ft}{s}$, $(v_r)_A = 0$

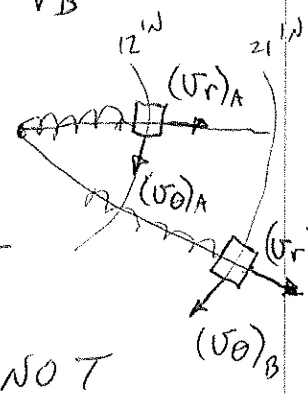
DETERMINE v_θ AND v_r WHEN $r_B = \frac{21}{12}\,ft$

CONSERVATION OF ANGULAR MOMENTUM: MOTION UNDER A CENTRAL FORCE

$$r_A m (v_\theta)_A = r_B m (v_\theta)_B \,,\quad (v_\theta)_B = \left(\frac{r_A}{r_B}\right)(v_\theta)_A$$

$$(v_\theta)_B = \left(\frac{12^{IN}}{21^{IN}}\right)\left(16\,\frac{ft}{s}\right) = \boxed{9.143\,\frac{ft}{s}}$$

CONSERVATION OF ENERGY: $T_A + V_A = T_B + V_B$



POINT A: $\Delta X_A = l_u - r_A = \frac{3}{2} - 1 = \frac{1}{2}\,ft$

$$(v_A)^2 = (v_r)_A^2 + (v_\theta)_A^2 = (v_\theta)_A^2$$

$$T_A = \frac{1}{2} m (v_\theta)_A^2 = \frac{1}{2}\frac{W}{g}(v_\theta)_A^2 \,,\quad V_A = \frac{1}{2}K(\Delta X_A)^2$$

POINT B: CASE 1: L.H.S. OF SPRING IS NOT ATTACHED TO POINT O:

SINCE $r_B = 21^{IN} > l_u = 18^{IN}$, $\Delta X_B = 0$ AND $V_B = 0$

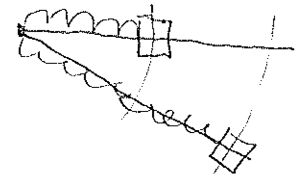
$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}\frac{W}{g}\left[(v_r)_B^2 + (v_\theta)_B^2\right]$$

$$\frac{1}{2}\frac{W}{g}(v_\theta)_A^2 + \frac{1}{2}K(\Delta X_A)^2 = \frac{1}{2}\frac{W}{g}\left[(v_r)_B^2 + (v_\theta)_B^2\right] + 0$$

$$(v_r)_B = \sqrt{(v_\theta)_A^2 - (v_\theta)_B^2 + \frac{gK}{W}(\Delta X_A)^2}$$

PROB. 13-95 CONT.

$$(v_r)_B = \sqrt{\left(16\,\frac{ft}{s}\right)^2 - \left(9.143\,\frac{ft}{s}\right)^2 + \frac{\left(32.2\,\frac{ft}{s^2}\right)\left(50\,\frac{lb}{ft}\right)}{(10\,lb)}\cdot\left(\frac{1}{2}\,ft\right)^2}$$



$$\boxed{(v_r)_B = 14.58\,\frac{ft}{s}}$$

POINT B: CASE 2: LHS OF SPRING IS ATTACHED TO POINT O!

$$\Delta X_B = r_B - l_u = \left(\frac{21}{12} - \frac{3}{2}\,ft\right) = \left(\frac{21-18}{12}\right) = \frac{3}{12} = \frac{1}{4}\,ft$$

$$V_B = \frac{1}{2}k(\Delta X_B)^2$$

$$\frac{1}{2}\frac{W}{g}(v_\theta)_A^2 + \frac{1}{2}k(\Delta X_A)^2 = \frac{1}{2}\frac{W}{g}\left[(v_r)_B^2 + (v_\theta)_B^2\right] + \frac{1}{2}k(\Delta X_B)^2$$

$$(v_r)_B = \sqrt{(v_\theta)_A^2 - (v_\theta)_B^2 + \frac{gk}{W}\left[(\Delta X_A)^2 - (\Delta X_B)^2\right]}$$

$$(v_r)_B = \sqrt{\left(16\,\frac{ft}{s}\right)^2 - \left(9.143\,\frac{ft}{s}\right)^2 + \frac{\left(50\,\frac{lb}{ft}\right)\left(32.2\,\frac{ft}{s^2}\right)}{(10\,lb)}\cdot\left[\left(\frac{1}{2}\,ft\right)^2 - \left(\frac{1}{4}\,ft\right)^2\right]}$$

$$\boxed{(v_r)_B = 14.23\,\frac{ft}{s}}$$