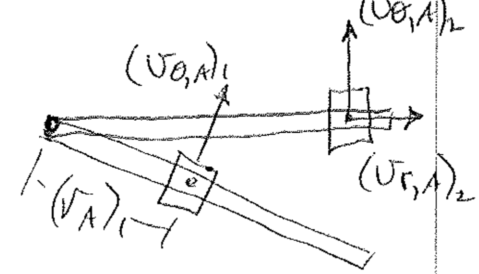
PROB. 13-98

$m_A = 1.8\text{ kg},\ m_B = 0.7\text{ kg},\ (v_{\theta,A})_1 = 2.1\ \frac{m}{s},\ (v_B)_1 = 0,$

$(r_A)_1 = 0.1\text{ m}$



a) FIND $(v_A)_2$ WHEN $(r_A)_2 = 0.2\text{ m}$

CONSERVATION OF ANGULAR MOMENTUM:

$$(r_A)_1 \cdot m_A (v_{\theta,A})_1 = (r_A)_2 \cdot m_A (v_{\theta,A})_2$$

$$(v_{\theta,A})_2 = \frac{(r_A)_1}{(r_A)_2} \cdot (v_{\theta,A})_1 = \frac{(0.1\text{ m})}{(0.2\text{ m})} \cdot \left(2.1\ \frac{m}{s}\right) = 1.05\ \frac{m}{s}$$

CONSERVATION OF ENERGY:

$$T_1 + V_1 = T_2 + V_2$$

$$(T_A)_1 + (T_B)_1 + (V_A)_1 + (V_B)_1 = (T_A)_2 + (T_B)_2 + (V_A)_2 + (V_B)_2$$

CHOOSE DATUM OF MASS B: $(y_B)_1 = 0$

CHOOSE DATUM OF MASS A: $(y_A)_1 = 0$

$$(T_A)_1 = \frac{1}{2} m_A (v_A)_1^2 = \frac{1}{2} m_A \left[\cancel{(v_{r,A})_1^2}^{\,0} + (v_{\theta,A})_1^2\right] = \frac{1}{2} m_A (v_{\theta,A})_1^2$$

$(T_B)_1 = 0,\ (V_A)_1 = 0,\ (V_B)_1 = 0$

AT POINT 2: $(y_B)_2 = 0.1\text{ m},\ (y_A)_2 = 0$

$$(T_A)_2 = \frac{1}{2}\left[(v_{r,A})_2^2 + (v_{\theta,A})_2^2\right],\quad (T_B)_2 = \frac{1}{2} m_B (v_B)_2^2$$

FROM KINEMATICS, $(v_B)_2 = (v_{r,A})_2$

$$(T_B)_2 = \frac{1}{2} m_B (v_{r,A})_2^2,\quad (V_A)_2 = 0,\quad (V_B)_2 = m_B g\left[(r_A)_2 - (r_A)_1\right]$$

$$\frac{1}{2} m_A (v_{\theta,A})_1^2 + 0 + 0 + 0 = \frac{1}{2} m_A \left[(v_{r,A})_2^2 + (v_{\theta,A})_2^2\right]$$

$$+ \frac{1}{2} m_B (v_{r,A})_2^2 + 0 + m_B g\left[(r_A)_2 - (r_A)_1\right]$$

PROB. 13-98 CONT.

$$\frac{1}{2}(m_A+m_B)(v_{r,A})_2^2 = \frac{1}{2}m_A(v_{\theta,A})_1^2 - \frac{1}{2}m_A(v_{\theta,A})_2^2 - m_B g\left[(r_A)_2-(r_A)_1\right]$$

$$(v_{r,A})_2 = \sqrt{\frac{2}{(m_A+m_B)}\cdot\left\{\frac{1}{2}m_A\left[(v_{\theta,A})_1^2-(v_{\theta,A})_2^2\right]-m_B g\,(r_{A2}-r_{A1})\right\}}$$

$$(v_{r,A})_2 = \sqrt{\frac{2}{(1.8+0.7\text{ kg})}\cdot\left\{\frac{(1.8\text{ kg})}{2}\left[\left(2.1\tfrac{m}{s}\right)^2-\left(1.05\tfrac{m}{s}\right)^2\right]-(0.7\text{ kg})\left(9.81\tfrac{m}{s^2}\right)\left[0.2-0.1\text{ m}\right]\right\}}$$

$$\boxed{(v_{r,A})_2 = 1.353\ \tfrac{m}{s}}$$

$$(v_A)_2 = \sqrt{(v_{r,A})_2^2+(v_{\theta,A})_2^2} = \sqrt{(1.353)^2+(1.05)^2}$$

$$\boxed{(v_A)_2 = 1.713\ \tfrac{m}{s}}$$

b) FIND v_A WHEN $v_B = 0$

CONSERVATION OF ANGULAR MOMENTUM:

$$(r_A)_1 m_A (v_{\theta,A})_1 = (r_A)_2 m_A (v_{\theta,A})_2$$

$$(v_{\theta,A})_2 = \frac{(r_A)_1}{(r_A)_2}\cdot(v_{\theta,A})_1$$

$$(v_A)_2^2 = (\cancel{v_{r,A}})_2^{2} + (v_{\theta,A})_2^2 = (v_{\theta,A})_2^2$$

$$(v_A)_2^2 = \left[\frac{(r_A)_1}{(r_A)_2}\cdot(v_{\theta,A})_1\right]^2$$

PROB. 13-98 CONT.

CONSERVATION OF ENERGY:

$$(T_A)_1 + (T_B)_1 + (V_A)_1 + (V_B)_1 = (T_A)_2 + (T_B)_2 + (V_A)_2 + (V_B)_2$$

POINT 1: $(y_B)_1 = 0$, $(y_A)_1 = 0$, $(v_B)_1 = 0$

$$(v_A)_1^2 = \cancel{(v_{r,A})_1^2}^{\,0} + (v_{\theta,A})_1^2 = (v_{\theta,A})_1^2$$

$$(T_A)_1 = \frac{1}{2} m_A (v_A)_1^2 = \frac{1}{2} m_A (v_{\theta,A})_1^2, \quad (T_B)_1 = 0$$

$$(V_A)_1 = 0, \quad (V_B)_1 = 0$$

POINT 2: $(y_B)_2 = X$, $(y_A)_2 = 0$, $(v_B)_2 = 0$

$$(v_A)_2^2 = \cancel{(v_{r,A})_2^2}^{\,0} + (v_{\theta,A})_2^2 = (v_{\theta,A})_2^2$$

$$(r_A)_2 = (0.1 + X)$$

$$(T_A)_2 = \frac{1}{2} m_A (v_A)_2^2 = \frac{1}{2} m_A (v_{\theta,A})_2^2$$

$$(T_B)_2 = 0, \quad (V_A)_2 = 0, \quad (V_B)_2 = m_B g X$$

$$\frac{1}{2} m_A (v_{\theta,A})_1^2 + 0 + 0 + 0 = \frac{1}{2} m_A (v_{\theta,A})_2^2 + 0 + 0 + m_B g X$$

$$\frac{1}{2} m_A (v_{\theta,A})_1^2 = \frac{1}{2} m_A (v_{\theta,A})_2^2 + m_B g X$$

$$(v_{\theta,A})_1^2 = \left[\frac{(r_A)_1}{(0.1+X)} \cdot (v_{\theta,A})_1 \right]^2 + \frac{2 m_B}{m_A} \cdot g X$$

$$\left(2.1 \frac{m}{s}\right)^2 = \left[\frac{(0.1^m)}{(0.1+X)} \cdot \left(2.1 \frac{m}{s}\right) \right]^2 + \frac{2(0.7^{kg})}{(1.8^{kg})} \cdot \left(9.81 \frac{m}{s^2}\right) \cdot X$$

$$4.41 = \frac{0.0441}{(0.1+X)^2} + 7.63 X$$

SOLVE ITERATIVELY

$X = 0.5652^m$

PROB. 13-98 CONT.

$$(v_A)_2 = (v_{\theta,A})_2 = \frac{(r_A)_1}{(r_A)_2} \cdot (v_{\theta,A})_1$$

$$(v_A)_2 = \frac{(r_A)_1}{(0.1 + x)} \cdot (v_{\theta,A})_1$$

$$(v_A)_2 = \frac{(0.1\,\text{m})}{(0.1 + 0.5652\,\text{m})} \cdot \left(2.1\ \frac{\text{m}}{\text{s}}\right) \boxed{= 0.3157\ \frac{\text{m}}{\text{s}}}$$