

PROB. 13-100

$$v_i = 26.9 \frac{\text{km}}{\text{s}} = 26.9 \times 10^3 \frac{\text{m}}{\text{s}}$$

$$r_A = 350 \times 10^6 \text{ m}, \quad r_B = 100 \times 10^6 \text{ m}, \quad M_J = 319 M_E$$

FIND ΔU FOR ORBITAL INSERTION.

CONSERVATION OF ANGULAR MOMENTUM:

$$v_A m v_A = v_B m v_B \Rightarrow v_B = \left(\frac{r_A}{r_B} \right) v_A$$

CONSERVATION OF ENERGY:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m v_A^2 - \frac{GM_J m}{r_A} = \frac{1}{2} m v_B^2 - \frac{GM_J m}{r_B}$$

$$\frac{1}{2} v_A^2 - \frac{1}{2} \left[\left(\frac{r_A}{r_B} \right) v_A \right]^2 = GM_J \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$v_A^2 \left\{ \frac{1}{2} \left[1 - \left(\frac{r_A}{r_B} \right)^2 \right] \right\} = 319 GM_E \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$v_A = \sqrt{319 \cdot g R^2 \left(\frac{1}{r_A} - \frac{1}{r_B} \right) \cdot \left\{ \frac{1}{2} \left[1 - \left(\frac{r_A}{r_B} \right)^2 \right] \right\}^{-1}}$$

$$v_A = \sqrt{319 \left(9.81 \frac{\text{m}}{\text{s}^2} \right) \left(6.37 \times 10^6 \text{ m} \right)^2 \left[\frac{1}{(350 \times 10^6 \text{ m})} - \frac{1}{(100 \times 10^6 \text{ m})} \right] \cdot \left\{ \frac{1}{2} \left[1 - \left(\frac{350 \times 10^6 \text{ m}}{100 \times 10^6 \text{ m}} \right)^2 \right] \right\}^{-1}}$$

$$v_A = 1.270 \times 10^4 \frac{\text{m}}{\text{s}}$$

$$\Delta U = 26.9 \times 10^3 - 1.27 \times 10^4 \frac{\text{m}}{\text{s}} = 1.42 \times 10^4 \frac{\text{m}}{\text{s}} = \boxed{14.2 \frac{\text{km}}{\text{s}}}$$