PROB. 13-101

$h_A = 8$ km, $h_B = 140$ km, $R_m = 1740$ km, $M_m = 0.0123 M_E$

a) FIND v_A

$$r_A = \left(1740 + 8\ \text{km}\right)\left(\frac{1000\ \text{m}}{\text{km}}\right) = 1.748 \times 10^6\ \text{m}$$

$$r_B = \left(1740 + 140\ \text{km}\right)\left(\frac{1000\ \text{m}}{\text{km}}\right) = 1.88 \times 10^6\ \text{m}$$

CONSERVATION OF ANGULAR MOMENTUM:

$$r_A m v_A = r_B m v_B$$

$$v_B = \left(\frac{r_A}{r_B}\right) v_A$$

CONSERVATION OF ENERGY: $T_A + V_A = T_B + V_B$

$$\frac{1}{2} m v_A^2 - \frac{G M_m \cdot m}{r_A} = \frac{1}{2} m v_B^2 - \frac{G M_m \cdot m}{r_B}$$

$$v_A^2 - \left[\left(\frac{r_A}{r_B}\right) \cdot v_A\right]^2 = 2 G M_m \left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$G M_m = G\left(0.0123 M_E\right) = 0.0123 \cdot g R_E^2$$

$$v_A = \sqrt{2(0.0123) g R_E^2 \left(\frac{1}{r_A} - \frac{1}{r_B}\right)\left[1 - \left(\frac{r_A}{r_B}\right)^2\right]^{-1}}$$

$$v_A = \sqrt{2(0.0123)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(6.37 \times 10^6\ \text{m}\right)^2 \left[\left(\frac{1}{1.748 \times 10^6\ \text{m}}\right) - \left(\frac{1}{1.88 \times 10^6\ \text{m}}\right)\right] \cdot \left[1 - \left(\frac{1.748 \times 10^6\ \text{m}}{1.88 \times 10^6\ \text{m}}\right)^2\right]^{-1}}$$

$$\boxed{v_A = 1704\ \frac{\text{m}}{\text{s}}}$$

PROB. 13.101 CONT.

b) FIND Δv

LEM: $v_B = \left(\frac{r_A}{r_B}\right) v_A = \left(\frac{1.748 \times 10^6 \text{ m}}{1.88 \times 10^6 \text{ m}}\right)\left(1704 \frac{\text{m}}{\text{s}}\right) = 1584 \frac{\text{m}}{\text{s}}$

COMMAND MODUCE: FOR A CIRCULAR ORBIT,

$$v_B = \sqrt{\frac{GM_m}{r_B}} = \sqrt{\frac{G(0.0123 M_E)}{r_B}} = \sqrt{\frac{0.0123 g R_E^2}{r_B}}$$

$$v_B = \sqrt{\frac{0.0123\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(6.37 \times 10^6 \text{ m})^2}{(1.88 \times 10^6 \text{ m})}} = 1614 \frac{\text{m}}{\text{s}}$$

$$\Delta v = 1614 - 1584 = \boxed{29.79 \frac{\text{m}}{\text{s}}}$$