PROB. 13-156

$m_A = m_B = m, \quad e = 0, \quad \vec{v}_A = (v_A)\hat{i}, \quad \vec{v}_B = (-v_B)\hat{i}$

a) SHOW THAT $v_A' = v_B' = \frac{1}{2}(v_A - v_B)$

CONSERVE MOMENTUM

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}_A' + m_B \vec{v}_B'$$

$$v_A - v_B = v_A' + v_B'$$

COEFFICIENT OF RESTITUTION:

$$\vec{v}_B' - \vec{v}_A' = e(\vec{v}_A - \vec{v}_B) \Rightarrow v_B' = v_A'$$

$$v_A - v_B = v_A' + (v_A') \Rightarrow \boxed{v_A' = \frac{1}{2}(v_A - v_B)}$$

b) SHOW THAT $\Delta T = \frac{1}{4} m (v_A + v_B)^2$

$$(T_A + T_B) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{m}{2}(v_A^2 + v_B^2)$$

$$(T_A + T_B)' = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2 = \frac{m}{2}\left[(v_A')^2 + (v_B')^2\right]$$

$$= \frac{m}{2}\left\{\left[\frac{1}{2}(v_A - v_B)\right]^2 + \left[\frac{1}{2}(v_A - v_B)\right]^2\right\}$$

$$= \frac{m}{2} \cdot \frac{1}{4} \cdot 2 (v_A - v_B)^2 = \frac{m}{4}(v_A^2 - 2 v_A v_B + v_B^2)$$

$$\Delta T = (T_A + T_B) - (T_A - T_B)'$$

$$= \frac{m}{2}(v_A^2 + v_B^2) - \frac{m}{4}(v_A^2 - 2 v_A v_B + v_B^2)$$

$$= \frac{m}{4} v_A^2 + \frac{m}{4} v_B^2 + \frac{m}{2} v_A v_B$$

$$= \frac{m}{4}(v_A^2 + 2 v_A v_B + v_B^2)$$

$$\boxed{\Delta T = \frac{m}{4}(v_A + v_B)^2}$$