PROB. 14-10

$m_A = 3^{kg}$, $m_B = 4^{kg}$, $m_C = 5^{kg}$

a) FIND $\bar{r} = \frac{1}{m}\sum m_i \vec{r}_i$ POSITION OF MASS CENTER

$m = \sum m_i = 3+4+5 = 12^{kg}$

$m_A \vec{r}_A = (3)(1.2)\hat{i} + (3)(1.5)\hat{k} = (3.6)\hat{i} + (4.5)\hat{k}$ kg·m

$m_B \vec{r}_B = (4)(0.9)\hat{i} + (4)(1.2)\hat{j} + (4)(1.2)\hat{k}$

$m_B \vec{r}_B = (3.6)\hat{i} + (4.8)\hat{j} + (4.8)\hat{k}$ kg·m

$m_C \vec{r}_C = (5)(2.4)\hat{j} + (5)(1.8)\hat{k} = (12)\hat{j} + (9)\hat{k}$ kg·m

$\bar{r} = \left(\frac{1}{12}\right)\left[(7.2)\hat{i} + (16.8)\hat{j} + (18.3)\hat{k}\right]$

$$\boxed{\bar{r} = (0.6)\hat{i} + (1.4)\hat{j} + (1.525)\hat{k}\ m}$$

b) FIND LINEAR MOMENTUM

$\vec{L} = m\bar{v} = m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C$

$m_A\vec{v}_A = (3^{kg})\left[(-4)\hat{i} + (4)\hat{j} + (6)\hat{k}\right]$

$= (-12)\hat{i} + (12)\hat{j} + (18)\hat{k}\ \frac{kg\cdot m}{s}$

$m_B\vec{v}_B = (4^{kg})\left[(-6)\hat{i} + (8)\hat{j} + (4)\hat{k}\right]$

$= (-24)\hat{i} + (32)\hat{j} + (16)\hat{k}\ \frac{kg\cdot m}{s}$

$m_C\vec{v}_C = (5^{kg})\left[(2)\hat{i} + (-6)\hat{j} + (-4)\hat{k}\right]$

$= (10)\hat{i} + (-30)\hat{j} + (-20)\hat{k}\ \frac{kg\cdot m}{s}$

PROB. 14-10 CONT.

$$\vec{L} = (-26)\hat{i} + (14)\hat{j} + (14)\hat{k} \ \frac{kg \cdot m}{s} = m\vec{v}$$

c) FIND THE ANGULAR MOMENTUM ABOUT CENTROIDAL FRAME $\vec{H}_G = \Sigma (\vec{r}_{i/G} \times m_i \vec{v}_i)$

FIND POINTS OF INTEREST:

$A(1.2, 0, 1.5)^m$, $B(0.9, 1.2, 1.2)^m$, $C(0, 2.4, 1.8)^m$,

$G(0.6, 1.4, 1.525)^m$

$\vec{r}_{A/G}$: $dx = x_A - x_G = (1.2) - (0.6) = 0.6^m$

$dy = y_A - y_G = 0 - (1.4) = -1.4^m$

$dz = z_A - z_G = (1.5) - (1.525) = -0.025^m$

$\vec{r}_{A/G} = (0.6)\hat{i} + (-1.4)\hat{j} + (-0.025)\hat{k}^m$

$\vec{r}_{B/G}$: $dx = x_B - x_G = (0.9) - (0.6) = 0.3^m$

$dy = y_B - y_G = (1.2) - (1.4) = -0.2^m$

$dz = z_B - z_G = (1.2) - (1.525) = -0.325^m$

$\vec{r}_{B/G} = (0.3)\hat{i} + (-0.2)\hat{j} + (-0.325)\hat{k}^m$

$\vec{r}_{C/G}$: $dx = x_C - x_G = 0 - (0.6) = -0.6^m$

$dy = y_C - y_G = (2.4) - (1.4) = 1.0^m$

$dz = z_C - z_G = (1.8) - (1.525) = 0.275^m$

$\vec{r}_{C/G} = (-0.6)\hat{i} + (1.0)\hat{j} + (0.275)\hat{k}^m$

PROB. 14-10 CONT.

$$\vec{r}_{A/G} \times m_A \vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.6 & -1.4 & -0.025 \\ -12 & 12 & 18 \end{vmatrix}$$

$$= [(-1.4)(18) - (-0.025)(12)]\hat{i} - [(0.6)(18) - (-0.025)(-12)]\hat{j}$$
$$+ [(0.6)(12) - (-1.4)(-12)]\hat{k}$$

$$= (-24.9)\hat{i} + (-10.5)\hat{j} + (-9.6)\hat{k} \ \frac{kg \cdot m^2}{s}$$

$$\vec{r}_{B/G} \times m_B \vec{v}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.3 & -0.2 & -0.325 \\ -24 & 32 & 16 \end{vmatrix}$$

$$= [(-0.2)(16) - (-0.325)(32)]\hat{i} - [(0.3)(16) - (-0.325)(-24)]\hat{j}$$
$$+ [(0.3)(32) - (-0.2)(-24)]\hat{k}$$

$$= (7.2)\hat{i} + (3.0)\hat{j} + (4.8)\hat{k} \ \frac{kg \cdot m^2}{s}$$

$$\vec{r}_{C/G} \times m_C \vec{v}_C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.6 & 1.0 & 0.275 \\ 10 & -30 & -20 \end{vmatrix}$$

$$= [(1.0)(-20) - (0.275)(-30)]\hat{i} - [(-0.6)(-20) - (0.275)(10)]\hat{j}$$
$$+ [(-0.6)(-30) - (1.0)(10)]\hat{k}$$

PROB. 14-10 CONT.

$$\vec{r}_{C/G} \times m_C \vec{v}_C = (-11.75)\hat{i} + (-9.25)\hat{j} + (8.0)\hat{k} \quad \frac{kg \cdot m^2}{s}$$

$$\boxed{\vec{H}_G = (-29.45)\hat{i} + (-16.75)\hat{j} + (3.2)\hat{k} \quad \frac{kg \cdot m^2}{s}}$$

d) SHOW THAT $\vec{H}_O = \vec{r} \times m\vec{v} + \vec{H}_G$

$$\vec{r} \times m\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.6 & 1.4 & 1.525 \\ -26 & 14 & 14 \end{vmatrix}$$

$$= [(1.4)(14) - (1.525)(14)]\hat{i} - [(0.6)(14) - (1.525)(-26)]\hat{j}$$
$$+ [(0.6)(14) - (1.4)(-26)]\hat{k}$$

$$= (-1.75)\hat{i} + (-48.05)\hat{j} + (44.80)\hat{k} \quad \frac{kg \cdot m^2}{s}$$

$$\vec{r} \times m\vec{v} + \vec{H}_G = (-31.2)\hat{i} + (-64.8)\hat{j} + (48.0)\hat{k} \quad \frac{kg \cdot m^2}{s}$$

FROM PROB. 14-9,

$$\vec{H}_O = (-31.2)\hat{i} + (-64.8)\hat{j} + (48.0)\hat{k} \quad \checkmark$$