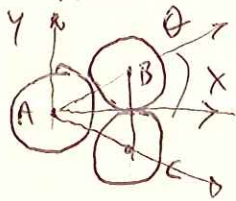


PROB. 14-38

$\vec{U}_0 = (U_0)\hat{i}$ ,  $e = 1$  PERFECTLY ELASTIC

FIND FINAL VELOCITIES WHEN

a) PATH OF A IS PERFECTLY CENTERED, A STRIKES B AND C SIMULTANEOUSLY



$\theta = 30^\circ$  EQUILATERAL TRIANGLE

CONSERVATION OF MOMENTUM:  $\vec{L}_1 = \vec{L}_2$

$$M\vec{U}_{A1} = M\vec{U}_{A2} + M\vec{U}_{B2} + M\vec{U}_{C2}$$

$$\vec{U}_{A2} = (U_{A2})\hat{i} \quad (\text{SYMMETRY})$$

$$\vec{U}_{B2} = (U_{B2} \cdot \cos 30^\circ)\hat{i} + (U_{B2} \cdot \sin 30^\circ)\hat{j}$$

$$\vec{U}_{B2} = (0.866 U_{B2})\hat{i} + (0.5 U_{B2})\hat{j}$$

$$\vec{U}_{C2} = (0.866 U_{C2})\hat{i} + (-0.5 U_{C2})\hat{j}$$

$$(U_0)\hat{i} = (U_{A2})\hat{i} + (0.866 U_{B2})\hat{i} + (0.5 U_{B2})\hat{j} \\ + (0.866 U_{C2})\hat{i} + (-0.5 U_{C2})\hat{j}$$

X-COMPONENT:

$$U_0 = U_{A2} + 0.866 U_{B2} + 0.866 U_{C2}$$

Y-COMPONENT:

$$0 = 0.5 U_{B2} - 0.5 U_{C2} \Rightarrow U_{B2} = U_{C2}$$

PROB. 14-38 CONT.

$$V_0 = V_{A2} + 1.732 V_{B2} \Rightarrow V_{A2} = V_0 - 1.732 V_{B2}$$

CONSERVATION OF ENERGY:

$$T_1 + V_1 = T_2 + V_2$$

$$V_1 = V_2 \quad \therefore \quad T_1 = T_2$$

$$T_1 = \frac{1}{2} M V_{A1}^2 = \frac{1}{2} M V_0^2$$

$$T_2 = \frac{1}{2} M V_{A2}^2 + \frac{1}{2} M V_{B2}^2 + \frac{1}{2} M V_{C2}^2 = \frac{1}{2} M V_{A2}^2 + M V_{B2}^2$$

$$\frac{1}{2} M V_0^2 = \frac{1}{2} M V_{A2}^2 + M V_{B2}^2$$

$$V_0^2 = V_{A2}^2 + 2 V_{B2}^2$$

$$V_0^2 = (V_0 - 1.732 V_{B2})^2 + 2 V_{B2}^2$$

$$V_0^2 = V_0^2 - 3.464 V_0 V_{B2} + 3 V_{B2}^2 + 2 V_{B2}^2$$

$$5 V_{B2} = 3.464 V_0 \Rightarrow \boxed{V_{B2} = 0.6928 V_0 = V_{C2}}$$

$$V_{A2} = V_0 - 1.732(0.6928 V_0) = \boxed{-0.2 V_0 \leftarrow}$$

b) NOT PERFECTLY CENTERED, A STRIKES B SLIGHTLY BEFORE IT STRIKES C.

FIRST IMPACT: A HITS B

CONSERVE MOMENTUM:

$$M \vec{V}_{A1} = M \vec{V}_{A2} + M \vec{V}_{B2}$$

$$(V_0) \hat{c} = (V_{Ax2}) \hat{c} + (V_{Ay2}) \hat{j} + (0.866 V_{B2}) \hat{c} + (0.5 V_{B2}) \hat{j}$$

PROB. 14-38 CONT.

X-DIRECTION:

$$V_0 = V_{Ax2} + 0.866 V_B \Rightarrow V_{Ax2} = V_0 - 0.866 V_B$$

$$V_{Ax2}^2 = (V_0 - 0.866 V_B)^2 = V_0^2 - 1.732 V_0 V_B + 0.75 V_B^2$$

Y-DIRECTION:

$$0 = V_{Ay2} + 0.5 V_B \Rightarrow V_{Ay2} = -0.5 V_B$$

$$V_{Ay2}^2 = (-0.5 V_B)^2 = 0.25 V_B^2$$

CONSERVE ENERGY:

$$\frac{1}{2} M V_0^2 = \frac{1}{2} M (V_{Ax2}^2 + V_{Ay2}^2) + \frac{1}{2} M V_B^2$$

$$V_0^2 = (V_0^2 - 1.732 V_0 V_B + 0.75 V_B^2) + (0.25 V_B^2) + V_B^2$$

$$0 = -1.732 V_0 V_B + 2 V_B^2 \Rightarrow \boxed{V_B = 0.866 V_0 \angle 30^\circ}$$

$$V_{Ax2} = V_0 - 0.866(0.866 V_0) = 0.25 V_0$$

$$V_{Ay2} = -0.5(0.866 V_0) = -0.433 V_0$$

$$\vec{V}_{A2} = (0.25 V_0) \hat{i} + (-0.433 V_0) \hat{j}$$

SECOND IMPACT: A HITS C

CONSERVE MOMENTUM:

$$M \vec{V}_{A1} = M \vec{V}_{A2} + M \vec{V}_{C2}$$

$$(0.25 V_0) \hat{i} + (-0.433 V_0) \hat{j} = (V_{Ax2}) \hat{i} + (V_{Ay2}) \hat{j} \\ + (0.866 V_{C2}) \hat{i} + (-0.5 V_{C2}) \hat{j}$$



PROB. 14-38 CONT.

X-DIRECTION:

$$0.25 U_0 = V_{Ax2} + 0.866 U_{c2}$$

$$V_{Ax2} = 0.25 U_0 - 0.866 U_{c2}$$

$$V_{Ax2}^2 = 0.0625 U_0^2 - 0.433 U_0 U_{c2} + 0.75 U_{c2}^2$$

Y-DIRECTION:

$$-0.433 U_0 = V_{Ay2} - 0.5 U_{c2}$$

$$V_{Ay2} = -0.433 U_0 + 0.5 U_{c2}$$

$$V_{Ay2}^2 = 0.1875 U_0^2 - 0.433 U_0 U_{c2} + 0.25 U_{c2}^2$$

CONSERVE ENERGY:

$$\frac{1}{2} m U_{A1}^2 = \frac{1}{2} m U_{A2}^2 + \frac{1}{2} m U_{c2}^2$$

$$U_{A1}^2 = (0.25 U_0)^2 + (-0.433 U_0)^2 = 0.25 U_0^2$$

$$0.25 U_0^2 = (0.0625 U_0^2 - 0.433 U_0 U_{c2} + 0.75 U_{c2}^2) + (0.1875 U_0^2 - 0.433 U_0 U_{c2} + 0.25 U_{c2}^2) + U_{c2}^2$$

$$0 = -0.866 U_0 U_{c2} + 2 U_{c2}^2 \Rightarrow \boxed{U_{c2} = 0.433 U_0 \rightarrow 30^\circ}$$

$$V_{Ax2} = 0.25 U_0 - 0.866 (0.433 U_0) = -0.125 U_0$$

$$V_{Ay2} = -0.433 U_0 + 0.5 (0.433 U_0) = -0.2165 U_0$$

$$\theta = \tan^{-1} \left( \frac{0.2165}{0.1125} \right) = 60^\circ$$

$$\boxed{\vec{V}_{A2} = 0.25 U_0 \rightarrow 60^\circ}$$