PROB. 14-45

$m = 360^{kg}$, $\vec{v}_0 = (450)\hat{k}\ \frac{m}{s}$, AT 0.

$m_A = 60^{kg}$, $m_B = 120^{kg}$, $m_C = 180^{kg}$

$A(72, 72, 648)^m$, $B(180, 396, 972)^m$, $C(-144, -288, 576)^m$

$\vec{v}_B = (150)\hat{i} + (330)\hat{j} + (660)\hat{k}\ \frac{m}{s}$

$\vec{v}_C = (-120)\hat{i} + (v_{Cy})\hat{j} + (v_{Cz})\hat{k}\ \frac{m}{s}$

$\vec{v}_A = (v_{Ax})\hat{i} + (v_{Ay})\hat{j} + (v_{Az})\hat{k}\ \frac{m}{s}$

FIND $\vec{v}_A$

CONSERVE MOMENTUM: NO EXTERNAL FORCES

$$m\vec{v}_0 = m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C$$

$$\vec{v}_A = \left(\frac{m}{m_A}\right)\vec{v}_0 - \left(\frac{m_B}{m_A}\right)\vec{v}_B - \left(\frac{m_C}{m_A}\right)\vec{v}_C$$

$$\vec{v}_A = \left(\frac{360}{60}\right)\vec{v}_0 - \left(\frac{120}{60}\right)\vec{v}_B - \left(\frac{180}{60}\right)\vec{v}_C$$

$$\vec{v}_A = 6\vec{v}_0 - 2\vec{v}_B - 3\vec{v}_C$$

$$(v_{Ax})\hat{i} + (v_{Ay})\hat{j} + (v_{Az})\hat{k} = (6v_0)\hat{k}$$

$$-2\left[(v_{Bx})\hat{i} + (v_{By})\hat{j} + (v_{Bz})\hat{k}\right]$$

$$-3\left[(v_{Cx})\hat{i} + (v_{Cy})\hat{j} + (v_{Cz})\hat{k}\right]$$

X-DIRECTION: $v_{Ax} = -2v_{Bx} - 3v_{Cx}$

PROB. 14-45 CONT.

$$\boxed{v_{Ax} = -2(150) - 3(-120) = 60 \ \frac{m}{s}}$$

Y-DIRECTION:

$$v_{Ay} = -2v_{By} - 3v_{Cy}$$

$$v_{Ay} = -2(330) - 3v_{Cy}$$

$$\boxed{v_{Ay} = -660 - 3v_{Cy}}$$

Z-DIRECTION:

$$v_{Az} = 6v_0 - 2v_{Bz} - 3v_{Cz}$$

$$v_{Az} = 6(450) - 2(660) - 3v_{Cz}$$

$$\boxed{v_{Az} = 1380 - 3v_{Cz}}$$

CONSERVE ANGULAR MOMENTUM ABOUT O:

$$(\vec{H}_O)_1 = (\vec{H}_O)_2$$

POSITION VECTORS:

$$\vec{r}_A = (72)\hat{i} + (72)\hat{j} + (648)\hat{k} \ m$$

$$\vec{r}_B = (180)\hat{i} + (396)\hat{j} + (972)\hat{k} \ m$$

$$\vec{r}_C = (-144)\hat{i} + (-288)\hat{j} + (576)\hat{k} \ m$$

PROB. 14-45 CONT.

$$(\vec{H}_O)_1 = \vec{r} \times m\vec{v}_0 = 0 \quad \text{SINCE } \vec{r} = 0$$

$$(\vec{H}_O)_2 = \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B + \vec{r}_C \times m_C \vec{v}_C$$

$$\vec{r}_A \times m_A \vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_{AX} & r_{AY} & r_{AZ} \\ m_A v_{AX} & m_A v_{AY} & m_A v_{AZ} \end{vmatrix}$$

$$= m_A \{ (r_{AY} v_{AZ} - r_{AZ} v_{AY}) \hat{i} - (r_{AX} v_{AZ} - r_{AZ} v_{AX}) \hat{j}$$
$$+ (r_{AX} v_{AY} - r_{AY} v_{AX}) \hat{k} \}$$

$$\vec{r}_B \times m_B \vec{v}_B = m_B \{ (r_{BY} v_{BZ} - r_{BZ} v_{BY}) \hat{i}$$
$$- (r_{BX} v_{BZ} - r_{BZ} v_{BX}) \hat{j}$$
$$+ (r_{BX} v_{BY} - r_{BY} v_{BX}) \hat{k} \}$$

$$\vec{r}_C \times m_C \vec{v}_C = m_C \{ (r_{CY} v_{CZ} - r_{CZ} v_{CY}) \hat{i}$$
$$- (r_{CX} v_{CZ} - r_{CZ} v_{CX}) \hat{j}$$
$$+ (r_{CX} v_{CY} - r_{CY} v_{CX}) \hat{k} \}$$

X-DIRECTION:

$$0 = m_A (r_{AY} v_{AZ} - r_{AZ} v_{AY}) + m_B (r_{BY} v_{BZ} - r_{BZ} v_{BY})$$
$$+ m_C (r_{CY} v_{CZ} - r_{CZ} v_{CY})$$

PROB. 14-45 CONT.

$$0 = (60)[(72)v_{Az} - (648)v_{Ay}]$$
$$+(120)[(396)(660) - (972)(330)]$$
$$+(180)[(-288)v_{Cz} - (576)v_{Cy}]$$

$$0 = 4320\,v_{Az} - 3.888\times10^4\,v_{Ay} - 7.128\times10^6$$
$$- 5.184\times10^4\,v_{Cz} - 1.0368\times10^5\,v_{Cy}$$

$$0 = 4320(1380 - 3\,v_{Cz}) - 3.888\times10^4(-660 - 3\,v_{Cy})$$
$$-7.128\times10^6 - 5.184\times10^4\,v_{Cz} - 1.0368\times10^5\,v_{Cy}$$

$$0 = -6.48\times10^4\,v_{Cz} + 1.296\times10^4\,v_{Cy} + 2.44944\times10^7$$

$$\boxed{v_{Cz} = 0.2\,v_{Cy} + 378}$$

Y-DIRECTION:

$$0 = -m_A(r_{Ax}v_{Az} - r_{Az}v_{Ax}) - m_B(r_{Bx}v_{Bz} - r_{Bz}v_{Bx})$$
$$-m_C(r_{Cx}v_{Cz} - r_{Cz}v_{Cx})$$

$$0 = (60)[(72)(1380 - 3v_{Cz}) - (648)(60)]$$
$$+(120)[(180)(660) - (972)(150)]$$
$$+(180)[(-144)v_{Cz} - (576)(-120)]$$

PROB. 14-45 CONT.

$$0 = 5.9616\times10^{6} - 1.296\times10^{4}\,v_{Cz} - 2.3328\times10^{6}$$

$$-3.24\times10^{6} - 2.592\times10^{4}\,v_{Cz} + 1.24416\times10^{7}$$

$$3.888\times10^{4}\,v_{Cz} = 1.28304\times10^{7}$$

$$\boxed{v_{Cz} = 330\ \tfrac{m}{s}}$$

$$\boxed{v_{Az} = 1380 - 3(330) = 390\ \tfrac{m}{s}}$$

$$v_{Cz} = 0.2\,v_{Cy} + 378$$

$$v_{Cy} = 5v_{Cz} - 1890 = 5(330) - 1890 \boxed{= -240\ \tfrac{m}{s}}$$

$$v_{Ay} = -660 - 3v_{Cy} = -660 - 3(-240) \boxed{= 60\ \tfrac{m}{s}}$$

$$\boxed{\vec{v}_A = (60)\,\hat{i} + (60)\,\hat{j} + (390)\,\hat{k}\ \tfrac{m}{s}}$$