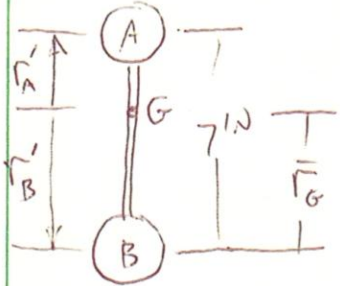
PROB. 14-47

$W_A = 5^{LB}, \quad W_B = 2^{LB}, \quad \vec{v}_{A,0} = (10.5)\hat{i} \ \frac{ft}{s}$

a) FIND $\vec{L}$ AND $\vec{H}_G$



$m\bar{r} = \Sigma m_i r_i$

MEASURING FROM B,

$(m_A + m_B)\bar{r}_G = m_A r_A + m_B \cancel{r_B}^{\,0}$

$\bar{r}_G = \left(\frac{m_A}{m_A + m_B}\right) r_A = \left(\frac{W_A}{W_A + W_B}\right) r_A$

$\bar{r}_G = \left(\frac{5}{5+2}\right)(7^{IN}) = 5^{IN}$

LINEAR MOMENTUM: $\vec{L} = \Sigma m_i \vec{v}_i = (m_A v_A)\hat{i} + (m_B \cancel{v_B}^{\,0})\hat{i}$

$\vec{L} = \left[\left(\frac{W_A}{g}\right) v_A\right]\hat{i} = \left[\left(\frac{5^{LB}}{32.2 \ ft/s^2}\right)\left(10.5 \ \frac{ft}{s}\right)\right]\hat{i} \ \boxed{= (1.630)\hat{i} \ LB \cdot s}$

ANGULAR MOMENTUM: $\vec{H}_G = \Sigma(\vec{r}_i' \times m_i \vec{v}_i)$

$\vec{H}_G = \vec{r}_A' \times m_A \vec{v}_A + \vec{r}_B' \times \cancel{m_B \vec{v}_B}^{\,0}$

$$\vec{H}_G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & r_A' & 0 \\ m_A v_A & 0 & 0 \end{vmatrix}$$

$\vec{H}_G = [0 - r_A' m_A v_A]\hat{k} = \left[-\left(\frac{2}{12} \ ft\right)\left(\frac{5^{LB}}{32.2 \ ft/s^2}\right)\left(10.5 \ \frac{ft}{s}\right)\right]\hat{k}$

$\boxed{\vec{H}_G = (-0.2717)\hat{k} \ ft\text{-}LB \cdot s}$

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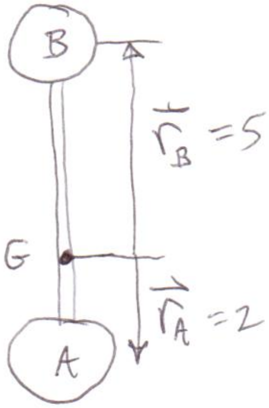
PROB. 14-47 CONT.

b) FIND v_A AND v_B AFTER AB ROTATES THROUGH 180°.

CONSERVE LINEAR MOMENTUM:



$$\vec{L}_1 = \vec{L}_2 = \Sigma(m_i \vec{v}_i)_2$$

$$\vec{v}_A = (v_{Ax})\hat{i} + (v_{Ay})\hat{j}$$

$$\vec{v}_B = (v_{Bx})\hat{i} + (v_{By})\hat{j}$$

$$\vec{L}_2 = m_A[(v_{Ax})\hat{i} + (v_{Ay})\hat{j}] + m_B[(v_{Bx})\hat{i} + (v_{By})\hat{j}]$$

$\vec{L}_1 = \vec{L}_2$:

X-DIRECTION:

$$m_A v_{A1} = m_A v_{Ax} + m_B v_{Bx} \quad (1)$$

Y-DIRECTION:

$$0 = m_A v_{Ay} + m_B v_{By} \quad (2)$$

CONSERVE ANGULAR MOMENTUM:

$$\vec{H}_{G1} = \vec{H}_{G2} = \Sigma(\vec{r}_i \times m_i v_i)_2$$

$$\vec{H}_{G2} = \vec{r}_B \times m_B \vec{v}_B + \vec{r}_A \times m_A \vec{v}_A$$

$$\vec{r}_B \times m_B \vec{v}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & r_B & 0 \\ m_B v_{Bx} & m_B v_{By} & 0 \end{vmatrix}$$

$$= [0 - m_B r_B v_{Bx}]\hat{k} = (-m_B r_B v_{Bx})\hat{k}$$

$$\vec{r}_A \times m_A \vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -r_A & 0 \\ m_A v_{Ax} & m_A v_{Ay} & 0 \end{vmatrix}$$

$$= [0 - m_A(-r_A) v_{Ax}]\hat{k} = (m_A r_A v_{Ax})\hat{k}$$

$\vec{H}_{G1} = \vec{H}_{G2}$:

$$-r_A m_A v_{A1} = -r_B m_B v_{Bx} + r_A m_A v_{Ax} \qquad (3)$$

CONSERVE ENERGY:

$$T_1 = T_2$$

$$T_1 = \frac{1}{2}\left(\sum m_i v_i^2\right), \quad T_2 = \frac{1}{2}\left(\sum m_i v_i^2\right)$$

$$T_1 = \frac{1}{2}\left(m_A v_{A1}^2 + m_B \cancel{v_{B1}^2}^{\,0}\right) = \frac{1}{2} m_A v_{A1}^2$$

$$T_2 = \frac{1}{2}\left(m_A v_{A2}^2 + m_B v_{B2}^2\right)$$

$$T_2 = \frac{1}{2}\left[m_A\left(v_{Ax}^2 + v_{Ay}^2\right) + m_B\left(v_{Bx}^2 + v_{By}^2\right)\right]$$

$T_1 = T_2$:

$$\frac{1}{2} m_A v_{A1}^2 = \frac{1}{2}\left[m_A\left(v_{Ax}^2 + v_{Ay}^2\right) + m_B\left(v_{Bx}^2 + v_{By}^2\right)\right]$$

$$m_A v_{A1}^2 = m_A\left(v_{Ax}^2 + v_{Ay}^2\right) + m_B\left(v_{Bx}^2 + v_{By}^2\right) \quad (4)$$

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(1): $v_{Ax} = v_{A1} - \left(\frac{m_B}{m_A}\right) v_{Bx}$

(1) INTO (3):

$$-r_A m_A v_{A1} = -r_B m_B v_{Bx} + \left(r_A m_A\right)\left[v_{A1} - \left(\frac{m_B}{m_A}\right) v_{Bx}\right]$$

$$-r_A m_A v_{A1} = -r_B m_B v_{Bx} + r_A m_A v_{A1} - r_A m_B v_{Bx}$$

$$\left(r_B m_B + r_A m_B\right) v_{Bx} = r_A m_A v_{A1} + r_A m_A v_{A1}$$

$$m_B\left(r_B + r_A\right) v_{Bx} = 2 r_A m_A v_{A1}$$

$$v_{Bx} = \frac{2 r_A m_A v_{A1}}{m_B\left(r_B + r_A\right)} = \frac{2 r_A W_A v_{A1}}{W_B\left(r_B + r_A\right)}$$

$$\boxed{v_{Bx}} = \frac{2\left(2^{\text{IN}}\right)\left(5^{\text{LB}}\right)\left(10.5\right)\frac{\text{ft}}{\text{s}}}{\left(2^{\text{LB}}\right)\left(5 + 2^{\text{IN}}\right)} \boxed{= 15\ \frac{\text{ft}}{\text{s}} \longrightarrow}$$

1 : $v_{Ax} = v_{A1} - \left(\frac{m_B}{m_A}\right) v_{Bx} = v_{A1} - \left(\frac{W_B}{W_A}\right) v_{Bx}$

$v_{Ax} = \left(10.5 \frac{ft}{s}\right) - \left(\frac{2}{5}\right)\left(15 \frac{ft}{s}\right)$

$$\boxed{v_{Ax} = 4.5 \frac{ft}{s} \rightarrow}$$

2 : $v_{Ay} = -\left(\frac{m_B}{m_A}\right) v_{By} = -\left(\frac{W_B}{W_A}\right) v_{By} = -\frac{2}{5} v_{By}$

~~3 and~~ 4 : $W_A v_{A1}^2 = W_A\left(v_{Ax}^2 + v_{Ay}^2\right) + W_B\left(v_{Bx}^2 + v_{By}^2\right)$

~~$m_A v_{A1}^2 = m_A$~~

$(5)(10.5)^2 = 5\left[(4.5)^2 + \left(-\frac{2}{5} v_{By}\right)^2\right] + (2)\left[(15)^2 + v_{By}^2\right]$

$551.25 = 101.25 + 0.8 v_{By}^2 + 450 + 2 v_{By}^2$

$2.8 v_{By}^2 = 0$

$$\boxed{v_{By} = 0}$$

$$\boxed{v_{Ay} = \frac{2}{5} v_{By} = 0}$$

$$\boxed{\begin{aligned} \vec{v}_A &= (4.5)\,\hat{i} \ \frac{ft}{s} \\ \vec{v}_B &= (15)\,\hat{i} \ \frac{ft}{s} \end{aligned}}$$