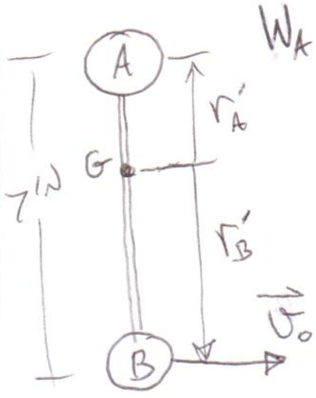
PROB. 14-48



$W_A = 5^{LB}$, $W_B = 2^{LB}$, $\vec{v}_{B,0} = (10.5)\hat{i}\ \frac{ft}{s}$

a) FIND $\vec{L}$ AND $\vec{H}_G$, T

$$m\bar{r} = \sum m_i r_i$$

MEASURING FROM B,

$$(m_A + m_B)\bar{r} = m_A r_A + m_B \cancel{r_B}^{\,0}$$

$$\bar{r} = \frac{m_A r_A}{(m_A + m_B)} = \frac{W_A r_A}{(W_A + W_B)} = \frac{(5^{LB})(7^{IN})}{(5 + 2^{LB})} = 5^{IN}$$

LINEAR MOMENTUM: $\vec{L}_1 = \left(\sum m_i \vec{v}_i\right)_1 = (m_A \cancel{\vec{v}_{A1}}^{\,0}) + (m_B \vec{v}_{B1})$

$$\boxed{\vec{L}_1 =} \left(\frac{W_B}{g}\right)\vec{v}_{B1} = \left(\frac{2^{LB}}{32.2\ \frac{ft}{s^2}}\right)(10.5)\hat{i}\ \frac{ft}{s}\ \boxed{= (0.6522)\hat{i}\ ^{LB\text{-}S}}$$

ANGULAR MOMENTUM: $\vec{H}_{G1} = \sum(\vec{r}_i \times m_i \vec{v}_i)$

$$\vec{H}_{G1} = \vec{r}_A' \times m_A \cancel{\vec{v}_{A1}}^{\,0} + \vec{r}_B' \times m_B \vec{v}_{B1}$$

$$\vec{H}_{G1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -r_B & 0 \\ m_B v_{B0} & 0 & 0 \end{vmatrix}$$

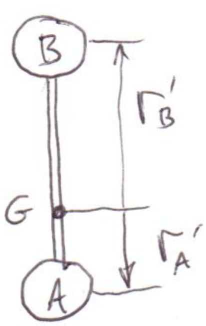
$$= [0 - (-r_B) m_B v_{B0}]\hat{k}$$

$$\boxed{\vec{H}_{G1}} = (r_B m_B v_{B0})\hat{k} = \left(\frac{5}{12}\,ft\right)\left(\frac{2\,lb}{32.2\,\frac{ft}{s^2}}\right)\left(10.5\,\frac{ft}{s}\right)\hat{k} = \boxed{(0.2717)\hat{k}\ ft\cdot lb\cdot s}$$

$$T_1 = \frac{1}{2}\left(\Sigma m_i v_i^2\right) = \frac{1}{2}\left(\cancel{m_A v_{A1}^2}^{\,0} + m_B v_{B1}^2\right)$$

$$\boxed{T_1} = \frac{1}{2}\left(\frac{2\,lb}{32.2\,\frac{ft}{s^2}}\right)\left(10.5\,\frac{ft}{s}\right)^2 \boxed{= 3.424\ ft\text{-}lb}$$

b) FIND $\vec{v}_A$ AND $\vec{v}_B$ AFTER AB ROTATES THROUGH 180°.



CONSERVE LINEAR MOMENTUM:

$$\vec{L}_1 = \vec{L}_2 = \Sigma(m_i \vec{v}_i)_2$$

$$\vec{v}_A = (v_{Ax})\hat{i} + (v_{Ay})\hat{j}$$

$$\vec{v}_B = (v_{Bx})\hat{i} + (v_{By})\hat{j}$$

$$\vec{L}_2 = m_A\left[(v_{Ax})\hat{i} + (v_{Ay})\hat{j}\right] + m_B\left[(v_{Bx})\hat{i} + (v_{By})\hat{j}\right]$$

$\vec{L}_1 = \vec{L}_2$:

X-DIRECTION:

$$m_B v_{B1} = m_A v_{Ax} + m_B v_{Bx} \qquad (1)$$

Y-DIRECTION:

$$0 = m_A v_{Ay} + m_B v_{By} \qquad (2)$$

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CONSERVE ANGULAR MOMENTUM:

$$\vec{H}_{G1} = \vec{H}_{G2} = \Sigma\left(\vec{r}_i \times m_i\vec{v}_i\right)_2$$

$$\vec{H}_{G2} = \vec{r}_A \times m_A\vec{v}_A + \vec{r}_B \times m_B\vec{v}_B$$

$$\vec{r}_A \times m_A\vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -r_A & 0 \\ m_A v_{Ax} & m_A v_{Ay} & 0 \end{vmatrix}$$

$$= \left[0 - (-r_A)(m_A v_{Ax})\right]\hat{k} = \left(r_A m_A v_{Ax}\right)\hat{k}$$

$$\vec{r}_B \times m_B\vec{v}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & r_B & 0 \\ m_B v_{Bx} & m_B v_{By} & 0 \end{vmatrix}$$

$$= \left[0 - r_B m_B v_{Bx}\right]\hat{k} = \left(-r_B m_B v_{Bx}\right)\hat{k}$$

$$\vec{H}_{G1} = \vec{H}_{G2} :$$

$$\left(r_B m_B v_{B\phi}\right) = r_A m_A v_{Ax} - r_B m_B v_{Bx} \quad (3)$$

CONSERVE ENERGY:

$$T_1 = T_2 = \frac{1}{2}\left(\Sigma m_i v_i^2\right)_2$$

$$T_2 = \frac{1}{2}\left(m_A v_{A2}^2 + m_B v_{B2}^2\right)$$

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$T_1 = T_2$:

$$\frac{1}{2} m_B v_{B1}^2 = \frac{1}{2}\left(m_A v_{A2}^2 + m_B v_{B2}^2\right)$$

$$\cancel{g}\, W_B v_{B1}^2 = W_A v_{A2}^2 + W_B v_{B2}^2 \qquad 4$$

$$1: \; v_{Ax} = \left(\frac{m_B}{m_A}\right) v_{B1} - \left(\frac{m_B}{m_A}\right) v_{Bx}$$

$$\cancel{v_{Ax} = \left(\frac{W_B}{W_A}\right) v_{B1} - \left(\frac{W_B}{W_A}\right) v_{Bx}}$$

1 INTO 3:

$$r_B m_B v_{B\phi} = r_A m_A \left[\left(\frac{m_B}{m_A}\right) v_{B1} - \left(\frac{m_B}{m_A}\right) v_{Bx}\right] - r_B m_B v_{Bx}$$

$$r_B m_B v_{B\phi} = r_A m_B v_{B1} - r_A m_B v_{Bx} - r_B m_B v_{Bx}$$

$$v_{Bx}\left(r_A m_B + r_B m_B\right) = r_A m_B v_{B1} - r_B m_B v_{B1}$$

$$v_{Bx} = \frac{m_B v_{B1}(r_A - r_B)}{m_B(r_A + r_B)} = \frac{v_{B1}(r_A - r_B)}{(r_A + r_B)}$$

$$\boxed{v_{Bx} =} \frac{\left(10.5 \frac{ft}{s}\right)(2 - 5 \text{ IN})}{(2 + 5 \text{ IN})} \boxed{= -4.5 \frac{ft}{s} \leftarrow}$$

$$\boxed{v_{Ax} =} \left(\frac{2}{5}\right)(10.5) - \left(\frac{2}{5}\right)(-4.5) \boxed{= 6 \frac{ft}{s} \rightarrow}$$

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2 : $v_{Ay} = -\left(\frac{m_B}{m_A}\right) v_{By} = -\left(\frac{2}{5}\right) v_{By}$

2 INTO 4 :

$$m_B v_{B1}^2 = m_A\left(v_{Ax}^2 + v_{Ay}^2\right) + m_B\left(v_{Bx}^2 + v_{By}^2\right)$$

$$W_B v_{B1}^2 = W_A\left(v_{Ax}^2 + v_{Ay}^2\right) + W_B\left(v_{Bx}^2 + v_{By}^2\right)$$

$$(2)(10.5)^2 = (5)\left[(6)^2 + \left\{-\tfrac{2}{5} v_{By}\right\}^2\right] + (2)\left[(-4.5)^2 + v_{By}^2\right]$$

$$220.5 = 180 + 0.8 v_{By}^2 + 40.5 + 2 v_{By}^2$$

$\boxed{v_{By} = 0}$

$\boxed{v_{Ay} = 0}$

$\boxed{\vec{v}_A = (6)\,\hat{i}\ \frac{ft}{s}}$

$\boxed{\vec{v}_B = (-4.5)\,\hat{i}\ \frac{ft}{s}}$