PROB. 14-51

$m_A = 3^{kg}$, $m_B = 1.5^{kg}$, $L_{CORD} = 0.6^m$

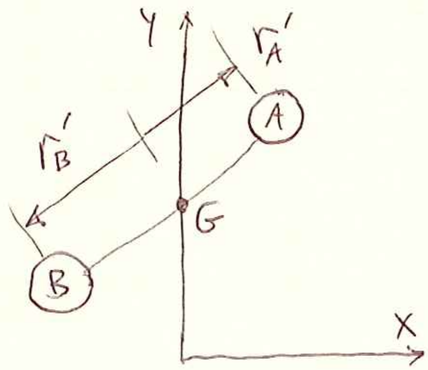
$\vec{\omega}_G = (10)\hat{k}\ \frac{RAD}{s}$; AT $t = 0$: $\bar{X}_0 = 0$, $\bar{Y}_0 = 2^m$

$\vec{v}_0 = (1.2)\hat{i} + (0.96)\hat{j}\ \frac{m}{s}$, $\vec{v}_A' = (v_A')\hat{j}$,

$\vec{v}_B' = (v_{BX}')\hat{i} + (v_{BY}')\hat{j}$, $b = 7.5^m$

a) FIND v_A' AND v_B'

MASS CENTER: $m\bar{r} = \sum m_i r_i = m_A r_A + m_B r_B$



MEASURED FROM G,

$\cancel{m\bar{r}}^{\,0} = m_A r_A - m_B r_B$

$r_A = \left(\frac{m_B}{m_A}\right) r_B$

$r_A + r_B = 0.6$, $r_B = 0.6 - r_A$

$r_A = \left(\frac{1.5}{3}\right)(0.6 - r_A)$, $r_A = 0.3 - \frac{1}{2} r_A$, $\frac{3}{2} r_A = 0.3$

$\boxed{r_A = 0.2^m}$ $\boxed{r_B = 0.6 - (0.2) = 0.4^m}$

LINEAR MOMENTUM OF MASS CENTER AT POINT X_0, Y_0:

$\vec{L}_0 = m\bar{v}_0 = (3 + 1.5^{kg})\left[(1.2)\hat{i} + (0.96)\hat{j}\right]\frac{m}{s}$

$\boxed{\vec{L}_0 = (5.4)\hat{i} + (4.32)\hat{j}\ \frac{kg \cdot m}{s}}$

ANGULAR MOMENTUM ABOUT MASS CENTER G AT X_0, Y_0:

$(\vec{H}_G)_0 = \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B$

PROB. 14-51 CONT.

$$v_A = r_A \omega \quad , \quad v_B = r_B \omega$$

$$(\vec{H}_G)_0 = [r_A m_A (r_A \omega)]\hat{k} + [r_B m_B (r_B \omega)]\hat{k}$$

$$(\vec{H}_G)_0 = (m_A \omega r_A^2)\hat{k} + (m_B \omega r_B^2)\hat{k}$$

$$(\vec{H}_G)_0 = [(3\text{ kg})(10\tfrac{\text{RAD}}{\text{s}})(0.2\text{ m})^2]\hat{k} + [(1.5\text{ kg})(10\tfrac{\text{RAD}}{\text{s}})(0.4\text{ m})^2]\hat{k}$$

$$\boxed{(\vec{H}_G)_0 = (3.6)\hat{k}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

ANGULAR MOMENTUM ABOUT THE ORIGIN O:

FROM PROB. 14-27,

$$\vec{H}_O = \bar{r} \times m\bar{v} + \vec{H}_G$$

$$\bar{r} \times m\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ (4.5)(1.2) & (4.5)(0.96) & 0 \end{vmatrix}$$

$$= [0 - (2\text{ m})(4.5\text{ kg})(1.2\tfrac{\text{m}}{\text{s}})]\hat{k} = (-10.8)\hat{k}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\vec{H}_O = (-10.8)\hat{k} + (3.6)\hat{k} \boxed{= (-7.2)\hat{k}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

KINETIC ENERGY AT X_0, Y_0:

$$T_0 = \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\Sigma m_i (v_i')^2 \qquad \text{EQN. 14.29}$$

$$T_0 = \tfrac{1}{2}(m_A + m_B)\bar{v}_0^2 + \tfrac{1}{2}[m_A (v_A')^2 + m_B (v_B')^2]$$

$$v_A' = r_A \omega \quad , \quad v_B' = r_B \omega$$

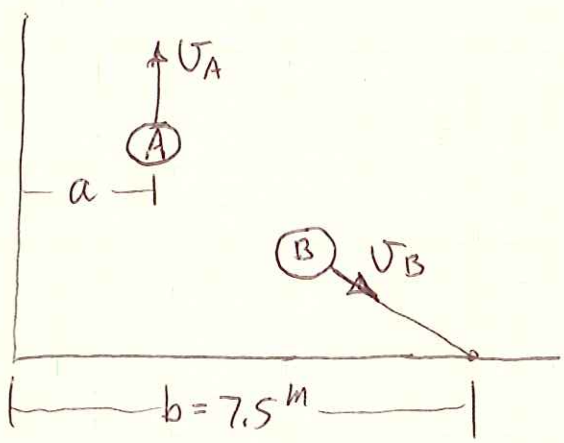
PROB. 14-51 CONT.

$$T_0 = \frac{1}{2}\left[(m_A+m_B)\bar{v}_0^{\,2} + m_A(r_A\omega)^2 + m_B(r_B\omega)^2\right]$$

$$T_0 = \frac{1}{2}\left\{(3+1.5\text{ kg})\left(1.2^2+0.96^2\ \frac{m^2}{s^2}\right) + (3\text{ kg})\left[(0.2\text{ m})\left(10\ \frac{\text{RAD}}{s}\right)\right]^2 + (1.5\text{ kg})\left[(0.4\text{ m})\left(10\ \frac{\text{RAD}}{s}\right)\right]^2\right\}$$

$$\boxed{T_0 = 23.31\text{ J}}$$

FINAL POSITIONS:



CONSERVE LINEAR MOMENTUM:

$$\vec{L}_0 = \vec{L} = \sum m_i \vec{v}_i$$

$$(5.4)\hat{i} + (4.32)\hat{j} = m_A\vec{v}_A + m_B\vec{v}_B$$

$$(5.4)\hat{i} + (4.32)\hat{j} = (3\text{ kg})(v_A)\hat{j} + (1.5\text{ kg})\left[(v_{Bx})\hat{i} + (v_{By})\hat{j}\right]$$

X-DIRECTION: $5.4 = 1.5\,v_{Bx} \Rightarrow v_{Bx} = 3.6\ \frac{m}{s}$

Y-DIRECTION: $4.32 = 3v_A + 1.5v_{By}$, $v_{By} = 2.88 - 2v_A$

CONSERVE ENERGY: $T_0 = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2$

$$T_0 = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_B\left[(v_{Bx})^2 + (v_{By})^2\right]$$

$$T_0 = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_B(v_{Bx})^2 + \frac{1}{2}m_B(v_{By})^2$$

$$23.31 = \frac{1}{2}(3\text{ kg})v_A^2 + \frac{1}{2}(1.5\text{ kg})\left(3.6\ \frac{m}{s}\right)^2 + \frac{1}{2}(1.5\text{ kg})(2.88 - 2v_A)^2$$

PROB. 14-51 CONT.

$$1.5 v_A^2 - 13.59 + 0.75\left(8.294 - 11.52 v_A + 4 v_A^2\right) = 0$$

$$4.5 v_A^2 - 8.64 v_A - 7.369 = 0$$

$$v_A = \frac{-(-8.64) \pm \sqrt{8.64^2 - 4(4.5)(-7.369)}}{2(4.5)}$$

$$\boxed{v_A = 0.96 \pm 1.6 = 2.56 \tfrac{m}{s}} \quad \text{OR} \quad -0.64 \tfrac{m}{s}$$

$$v_{By} = 2.88 - 2(2.56) = -2.24 \tfrac{m}{s}$$

$$\vec{v}_B = (3.6)\hat{i} + (-2.24)\hat{j} \tfrac{m}{s}, \quad \theta = \tan^{-1}\left(\frac{2.24}{3.6}\right) = 31.89^\circ$$

$$\boxed{\vec{v}_B = 4.24 \tfrac{m}{s} \searrow 31.89^\circ}$$

CONSERVE ANGULAR MOMENTUM ABOUT ORIGIN O:

$$(\vec{H}_O)_0 = \vec{H}_O = \Sigma\left(\vec{r}_i \times m_i \vec{v}_i\right) = \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B$$

$$\vec{r}_A \times m_A \vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & 0 & 0 \\ 0 & (3)(2.56) & 0 \end{vmatrix} = (7.68a)\hat{k}$$

$$\vec{r}_B \times m_B \vec{v}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7.5 & 0 & 0 \\ (1.5)(3.6) & (1.5)(-2.24) & 0 \end{vmatrix} = (-25.2)\hat{k}$$

$$(-7.2)\hat{k} = (7.68a)\hat{k} + (-25.2)\hat{k} \Rightarrow \boxed{a = 2.344 \, m}$$