PROB. 14-52

$m_A = 2^{kg}$, $m_B = 1^{kg}$, $X_0 = 0$, $Y_0 = 1.89^m$

$\vec{v}_A = (5)\hat{j}\ \frac{m}{s}$, $a = 2.56^m$, $\vec{v}_B = (7.2)\hat{i} + (-4.6)\hat{j}\ \frac{m}{s}$

$b = 7.48^m$

FIND $\bar{v}_0$, L AND $\omega$

MASS CENTER MEASURED FROM G:

$\cancel{m\bar{r}}^{\,0} = m_A r_A' + m_B r_B'$

$r_A = \left(\frac{m_B}{m_A}\right) r_B = \frac{1}{2} r_B$

$r_A + r_B = L$

$\left(\frac{1}{2} r_B\right) + r_B = L \Rightarrow r_B = \frac{2}{3}L$, $r_A = \frac{1}{3}L$

LINEAR MOMENTUM OF MASS CENTER AT $X_0, Y_0$:

$\vec{L}_0 = m\bar{v}_0 = (2+1^{kg})\bar{v}_0 = 3\bar{v}_0$

ANGULAR MOMENTUM OF MASS CENTER AT $X_0, Y_0$:

$(\vec{H}_G)_0 = \vec{r}_A \times m_A\vec{v}_A + \vec{r}_B \times m_B\vec{v}_B$

$v_A = r_A\omega$, $v_B = r_B\omega$

$(\vec{H}_G)_0 = [r_A m_A (r_A\omega)]\hat{k} + [r_B m_B (r_B\omega)]\hat{k}$

$(\vec{H}_G)_0 = [\omega(m_A r_A^2 + m_B r_B^2)]\hat{k}$

$(\vec{H}_G)_0 = [\omega\{(2)(\frac{1}{3}L)^2 + (1)(\frac{2}{3}L)^2\}]\hat{k}$

PROB. 14-52 CONT.

$$(\vec{H}_G)_0 = \left(\frac{2}{3}\omega L^2\right)\hat{k}$$

ANGULAR MOMENTUM ABOUT THE ORIGIN O : PROB. 14-27

$$\vec{H}_0 = \bar{r} \times m\bar{v} + \vec{H}_G$$

$$\vec{r} \times m\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.89 & 0 \\ 3v_{0x} & 3v_{0y} & 0 \end{vmatrix}$$

$$= \left[0 - (3)(1.89)v_{0x}\right]\hat{k} = (-5.67\,v_{0x})\hat{k}$$

$$\vec{H}_0 = (-5.67\,v_{0x})\hat{k} + \left(\frac{2}{3}\omega L^2\right)\hat{k}$$

$$\boxed{(\vec{H}_0)_0 = \left(\frac{2}{3}\omega L^2 - 5.67\,v_{0x}\right)\hat{k}}$$

KINETIC ENERGY AT $X_0, Y_0$ : EQN. 14.29

$$T_0 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}\sum m_i (v_i')^2$$

$$T_0 = \frac{1}{2}(m_A + m_B)v_0^2 + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$v_A = r_A\omega = \left(\frac{L}{3}\omega\right) = \frac{1}{3}L\omega, \quad v_B = r_B\omega = \frac{2}{3}L\omega$$

$$T_0 = \frac{1}{2}(3^{kg})v_0^2 + \frac{1}{2}(2^{kg})\left(\frac{1}{3}L\omega\right)^2 + \frac{1}{2}(1^{kg})\left(\frac{2}{3}L\omega\right)^2$$

$$T_0 = \frac{3}{2}v_0^2 + \frac{1}{9}L^2\omega^2 + \frac{2}{9}L^2\omega^2$$

$$\boxed{T_0 = \frac{3}{2}v_0^2 + \frac{1}{3}L^2\omega^2}$$

PROB. 14-52 CONT.

CONSERVE LINEAR MOMENTUM: $m\vec{v} = \Sigma m_i \vec{v}_i$

$$(m_A + m_B)\vec{v}_0 = m_A \vec{v}_A + m_B \vec{v}_B$$

$$3\vec{v}_0 = (2^{kg})(5)\hat{j}\ \tfrac{m}{s} + (1^{kg})\left[(7.2)\hat{i} + (-4.6)\hat{j}\ \tfrac{m}{s}\right]$$

$$\boxed{\vec{v}_0 = (2.4)\hat{i} + (1.8)\hat{j}\ \tfrac{m}{s}}$$

CONSERVE ANGULAR MOMENTUM ABOUT O: $(\vec{H}_0)_0 = \vec{H}_0$

$$(\vec{H}_0)_0 = \left(\tfrac{2}{3}\omega L^2 - 5.67 v_{0x}\right)\hat{k} = \left[\tfrac{2}{3}\omega L^2 - 5.67(2.4)\right]\hat{k}$$

$$(\vec{H}_0)_0 = \left(\tfrac{2}{3}\omega L^2 - 13.61\right)\hat{k}$$

$$\vec{H}_0 = \Sigma(\vec{r}_i \times m_i \vec{v}_i) = \vec{r}_A \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B$$

$$\vec{r}_A \times m_A \vec{v}_A = (2.56^{m})(2^{kg})(5\ \tfrac{m}{s})\hat{k} = (25.6)\hat{k}\ \tfrac{kg \cdot m^2}{s}$$

$$\vec{r}_B \times m_B \vec{v}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7.48 & 0 & 0 \\ (1)(7.2) & (1)(-4.6) & 0 \end{vmatrix}$$

$$= \left[(7.48)(-4.6)\right]\hat{k} = (-34.41)\hat{k}\ \tfrac{kg \cdot m^2}{s}$$

$$\tfrac{2}{3}\omega L^2 - 13.61 = 25.6 - 34.41$$

$$\omega L^2 = 7.197 \Rightarrow L^2 = \frac{7.197}{\omega}$$

PROB. 14-52 CONT.

CONSERVE ENERGY: $T_0 = T$

$$T_0 = \frac{3}{2}v_0^2 + \frac{1}{3}L^2\omega^2$$

$$T = \frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$\frac{3}{2}(2.4^2 + 1.8^2) + \frac{1}{3}L^2\omega^2 = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)(7.2^2 + 4.6^2)$$

$$L^2\omega^2 = 144$$

$$\left(\frac{7.197}{\omega}\right)\omega^2 = 144$$

$$\boxed{\omega = 20\ \frac{\text{RAD}}{\text{S}}}$$

$$\boxed{L = \sqrt{\frac{7.197}{20}} = 0.6\text{ m}}$$

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