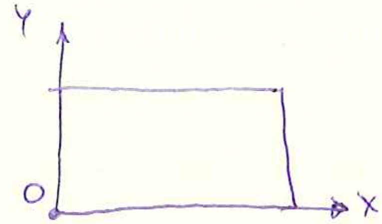
PROB. 14-54

$v_0 = 15 \frac{ft}{s}$, $v_C = 9.6 \frac{ft}{s}$, $C = 48^{IN}$

FIND v_A, v_B, a



CONSERVE LINEAR MOMENTUM:

$$\vec{v}_0 = \vec{v}_A + \vec{v}_B + \vec{v}_C$$

$$(15)\hat{i} = (-v_A)\hat{j} + (v_{Bx})\hat{i} + (v_{By})\hat{j} + (9.6)\hat{i}$$

X-DIRECTION: $15 = v_{Bx} + 9.6 \Rightarrow \boxed{v_{Bx} = 5.4 \frac{ft}{s}}$

Y-DIRECTION: $0 = -v_A + v_{By} \Rightarrow \boxed{v_{By} = v_A}$

CONSERVE ANGULAR MOMENTUM ABOUT ORIGIN:

$$(\vec{H}_0)_1 = (\vec{H}_0)_2$$

$$\vec{r}_A \times m_A \vec{v}_0 = \vec{r}_{A'} \times m_A \vec{v}_A + \vec{r}_B \times m_B \vec{v}_B + \vec{r}_C \times m_C \vec{v}_C$$

$$\vec{r}_A \times \vec{v}_0 = \vec{r}_{A'} \times \vec{v}_A + \vec{r}_B \times \vec{v}_B + \vec{r}_C \times \vec{v}_C$$

$$\vec{r}_A \times \vec{v}_0 = -(30^{IN})(15 \tfrac{ft}{s})\hat{k} = (-450)\hat{k} \ \tfrac{IN \cdot ft}{s}$$

$$\vec{r}_{A'} \times \vec{v}_A = -(a)(v_A)\hat{k} = (-a \cdot v_A)\hat{k} \ \tfrac{IN \cdot ft}{s}$$

$$\vec{r}_B \times \vec{v}_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 72.5 & 60 & 0 \\ 5.4 & v_A & 0 \end{vmatrix}$$

$$= [(72.5)v_A - (60)(5.4)]\hat{k} = (72.5 v_A - 324)\hat{k} \ \tfrac{IN \cdot FT}{s}$$

$$\vec{r}_C \times \vec{v}_C = -(60 - 48^{IN})(9.6 \tfrac{ft}{s}) = (-115.2)\hat{k} \ \tfrac{IN \cdot ft}{s}$$

PROB. 14-54 CONT.

$$-450 = -a \cdot v_A + 72.5 v_A - 324 - 115.2$$

$$\boxed{(72.5 - a) v_A = -10.8}$$

CONSERVE ENERGY: $\frac{1}{2} m v^2 = \frac{1}{2} m_i v_i^2$

$$m_A v_0^2 = m_A v_A^2 + m_B v_B^2 + m_C v_C^2$$

$$v_0^2 = v_A^2 + v_B^2 + v_C^2$$

$$v_0^2 = v_A^2 + v_{BX}^2 + v_{BY}^2 + v_C^2$$

$$(15)^2 = v_A^2 + (5.4)^2 + v_A^2 + (9.6)^2$$

$$\boxed{v_A = 7.2 \ \frac{ft}{s} \downarrow}$$

$$72.5 - a = -\frac{10.8}{7.2} \Rightarrow \boxed{a = 74.0 \text{ IN}}$$

$$\vec{v}_B = (5.4)\hat{i} + (7.2)\hat{j}, \quad \theta = \tan^{-1}\left(\frac{7.2}{5.4}\right) = 53.13^\circ$$

$$\boxed{\vec{v}_B = 9.0 \ \frac{ft}{s} \ \angle 53.13^\circ}$$

TOPS.
35500