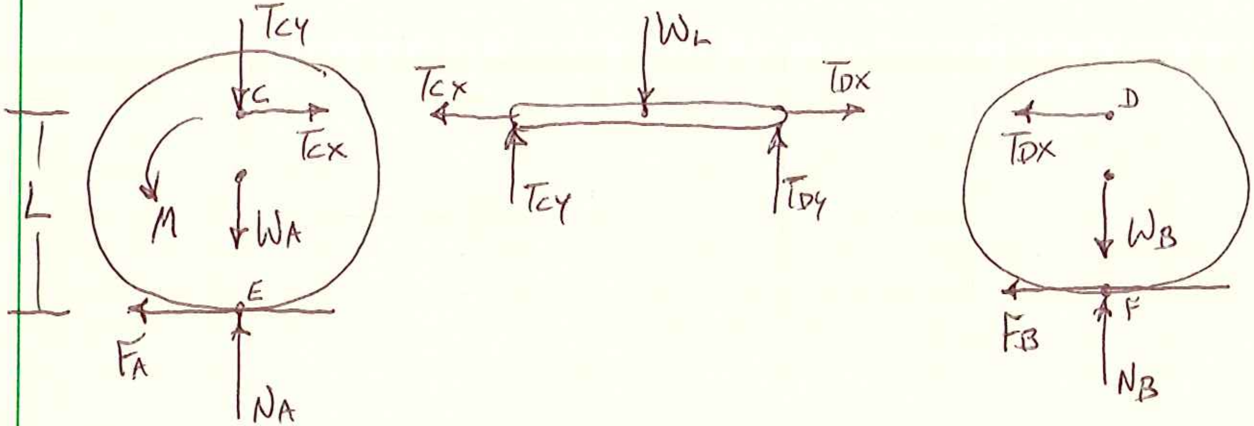
PROB. 16-109

$W_A = W_B = 4^{LB}$, $W_L = 3^{LB}$, $M = 1.5^{ft\text{-}LB}$, $r = \frac{1}{2}^{ft}$,

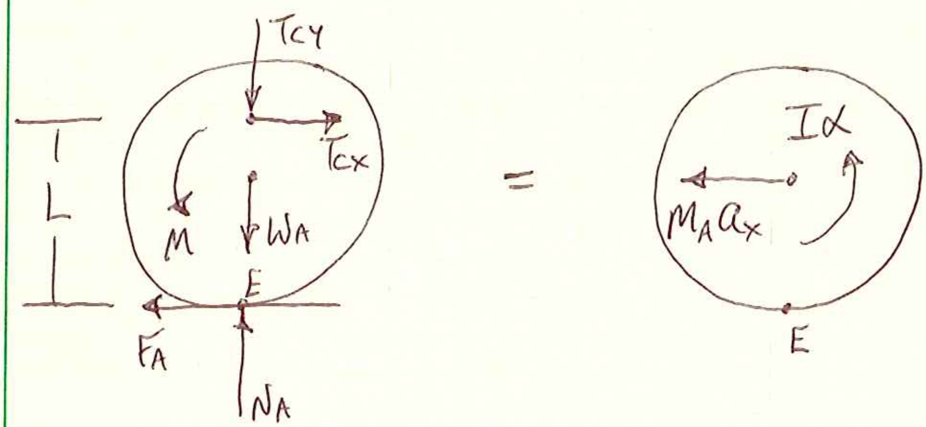
DISKS ROLL WITHOUT SLIDING, FIND a_A, a_B, T_{DX}

SINCE DISKS ROLL WITHOUT SLIDING, $a_{XA} = a_{XB} = r\alpha$

SINCE DISKS ARE LINKED TOGETHER, $a_{XA} = a_{XB} = a_X$



DISK A



$\sum M_E = \sum (M_E)_{EFF}$ $+\circlearrowleft$: $M - LT_{CX} = r_A M_A a_X + I\alpha$

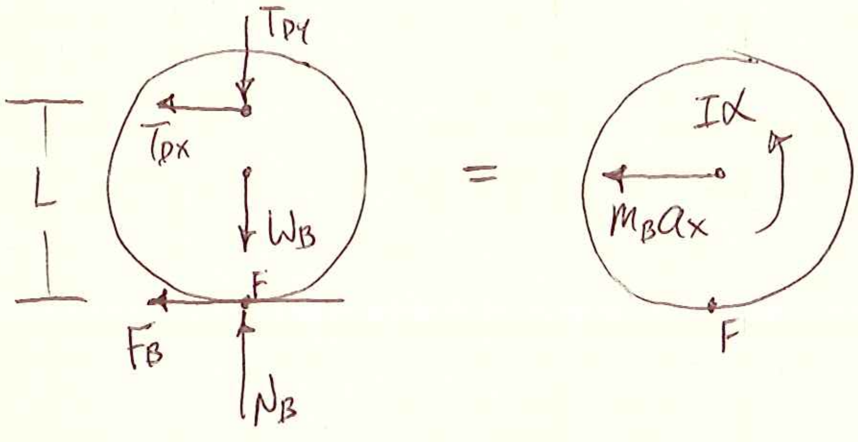
$M - LT_{CX} = r_A \left(\frac{W_A}{g}\right) r_A \alpha + \frac{1}{2} M_A r_A^2 \alpha$

$M - LT_{CX} = \frac{3}{2}\left(\frac{W_A}{g}\right) r_A^2 \alpha$

$$\boxed{T_{CX} = \frac{1}{L}\left[M - \frac{3}{2}\left(\frac{W_A}{g}\right) r_A^2 \alpha\right]}$$

PROB. 16-109 CONT.

DISK B

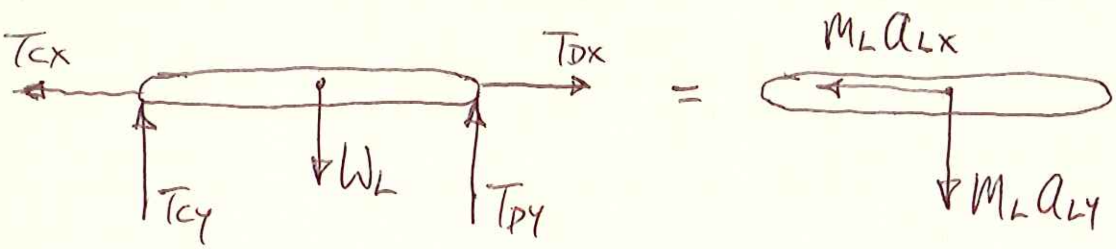


$$\Sigma M_F = \Sigma (M_F)_{EFF} \; +\circlearrowleft : \; L T_{DX} = r_B \cdot m_B a_X + I\alpha$$

$$L T_{DX} = r_B \left(\frac{W_B}{g}\right) r_B \alpha + \frac{1}{2}\left(\frac{W_B}{g}\right) r_B^2 \alpha$$

$$\boxed{T_{DX} = \frac{1}{L}\left[\frac{3}{2}\left(\frac{W_B}{g}\right) r_B^2 \alpha\right]}$$

LINK: TRANSLATION WITHOUT ROTATION



$$\Sigma F_X = m a_X : \; T_{DX} - T_{CX} = -m_L a_{LX}$$

$$\vec{a}_{LX} = \vec{a}_C = L\alpha$$

$$T_{DX} - T_{CX} = -\left(\frac{W_L}{g}\right) L\alpha$$

$$\frac{1}{L}\left[\frac{3}{2}\left(\frac{W_B}{g}\right) r_B^2 \alpha\right] - \frac{1}{L}\left[M - \frac{3}{2}\left(\frac{W_A}{g}\right) r_A^2 \alpha\right] = -\left(\frac{W_L}{g}\right) L\alpha$$

PROB. 16-109 CONT.

$$\frac{1}{L}\left[\frac{3}{2}\left(\frac{W_B}{g}\right)r_B^2\alpha\right]-\frac{M}{L}+\frac{1}{L}\left[\frac{3}{2}\left(\frac{W_A}{g}\right)r_A^2\alpha\right]=-\left(\frac{W_L}{g}\right)L\alpha$$

$W_A = W_B = W$ AND $r_A = r_B = r$

$$\frac{3}{L}\left(\frac{W}{g}\right)r^2\alpha-\frac{M}{L}=-\left(\frac{W_L}{g}\right)L\alpha$$

$$\alpha\left[\frac{3}{L}\left(\frac{W}{g}\right)r^2+\left(\frac{W_L}{g}\right)L\right]=\frac{M}{L}$$

$$\alpha=\frac{\left(\frac{M}{L}\right)}{\left[\frac{3}{L}\left(\frac{W}{g}\right)r^2+\left(\frac{W_L}{g}\right)L\right]}$$

$$\alpha=\frac{gM}{\left(3Wr^2+W_L L^2\right)}=\frac{\left(32.2\,\frac{ft}{s^2}\right)\left(1.5\,^{LB\cdot ft}\right)}{\left[3\left(4\,^{LB}\right)\left(\frac{1}{2}\,ft\right)^2+\left(3\,^{LB}\right)\left(\frac{8}{12}\,ft\right)^2\right]}$$

$$\boxed{\alpha = 11.15\ \frac{RAD}{s^2}}$$

$$a_A = a_B = r\alpha = \left(\frac{6}{12}\,ft\right)\left(11.15\ \frac{RAD}{s^2}\right)\boxed{=5.573\ \frac{ft}{s^2}}$$

$$T_{Dx}=\frac{3}{2}\cdot\frac{\left(4\,^{LB}\right)\left(\frac{1}{2}\,ft\right)^2\left(11.15\ \frac{RAD}{s^2}\right)}{\left(\frac{8}{12}\,ft\right)\left(32.2\,\frac{ft}{s^2}\right)}\boxed{=0.7788\,^{LB}}$$