PROB. 17-3

DISK A: $R_A = r$, $t_A = b$

DISK B: $R_B = nr$, $t_B = 3b$

$M = \text{CONSTANT}$, $\omega_0 = 0$, $\theta_1 = 0$, $\theta_2 = \left(2\,\text{REV}\right)\left(\frac{2\pi}{\text{REV}}\right) = 4\pi\ \text{RAD}$

FIND n FOR THE HIGHEST SPEED ON RIM OF DISK B.

WORK AND ENERGY: $T_1 + U_{1-2} = T_2$

KINETIC ENERGY: $T = \cancel{\frac{1}{2} m v^2}^{\,0} + \frac{1}{2} I \omega^2$

$T_1 = 0$ SINCE $\omega_0 = 0$, $T_2 = \frac{1}{2} I \omega_f^2$

$$U_{1-2} = \int_{\theta_1}^{\theta_2} M\, d\theta = M(\theta_2 - \theta_1)$$

$$0 + M(\theta_2 - \theta_1) = \frac{1}{2} I \omega^2$$

$$I = \frac{1}{2} m_A R_A^2 + \frac{1}{2} m_B R_B^2$$

$$m_A = \rho V_A = \rho(\pi R_A^2 t_A) = \pi \rho r^2 b$$

$$m_B = \rho V_B = \rho(\pi R_B^2 t_B) = \pi \rho (nr)^2 (3b) = 3\pi \rho n^2 r^2 b$$

$$I = \frac{1}{2}(\pi \rho r^2 b) r^2 + \frac{1}{2}(3\pi \rho n^2 r^2 b)(nr)^2$$

$$I = \frac{1}{2}\pi \rho r^4 b + \frac{3}{2}\pi \rho n^4 r^4 b = \frac{\pi}{2}\rho r^4 b\,(1 + 3n^4)$$

$$M(\theta_2 - \theta_1) = \frac{1}{2}\left[\frac{\pi}{2}\rho r^4 b (1 + 3n^4)\right]\omega^2$$

VELOCITY ON RIM OF DISK B:

$$v_B = R_B \omega_B = nr\omega, \quad \omega = \frac{v_B}{nr}$$

$$M(\theta_2-\theta_1) = \frac{\pi}{4}\rho r^4 b(1+3n^4)\left(\frac{v_B}{nr}\right)^2$$

$$v_B^2 = \frac{4}{\pi}\cdot\frac{M(\theta_2-\theta_1)}{\rho r^2 b}\cdot\left(\frac{n^2}{1+3n^4}\right)$$

TO MAXIMIZE v_B, TAKE DERIVATIVE W.R.T. n:

$$\frac{d}{dn}\left(\frac{n^2}{1+3n^4}\right) = 0$$

$$\frac{d}{dn}\left[n^2(1+3n^4)^{-1}\right] = n^2\cdot\left[(-1)(1+3n^4)^{-2}(12n^3)\right] + (1+3n^4)^{-1}\cdot 2n$$

$$= -\frac{12n^5}{(1+3n^4)^2} + \frac{2n}{(1+3n^4)} = 0$$

$$-12n^5 + 2n(1+3n^4) = 0$$

$$-12n^5 + 2n + 6n^5 = 0$$

$$-6n^5 + 2n = 0$$

$$n(2-6n^4) = 0$$

$$n = 0 \text{ AND } 2-6n^4 = 0 \Rightarrow \boxed{n = \left(\frac{1}{3}\right)^{1/4} = 0.7598}$$