PROB. 17-11

$W_P = 30^{LB}$, $K_P = 6.5^{IN}$, $W_A = 25^{LB}$, $W_B = 20^{LB}$

$\mu_K = 0.25$, $\omega_0 = 0$

a) FIND v_A WHEN BLOCK A HITS THE GROUND

WORK AND ENERGY: $T_1 + U_{1-2} = T_2$

KINETIC ENERGY: $T = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

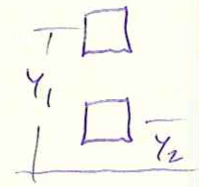
SYSTEM IS INITIALLY AT REST, SO $T_1 = 0$

WHEN BLOCK A HITS THE GROUND,

$T_{A2} = \frac{1}{2} m_A v_A^2 = \frac{1}{2}\left(\frac{W_A}{g}\right) v_A^2$

$T_{B2} = \frac{1}{2} m_B v_B^2 = \frac{1}{2}\left(\frac{W_B}{g}\right) v_B^2$

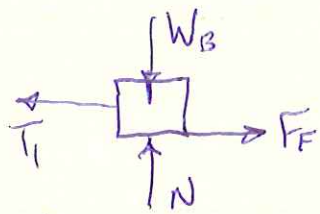
$T_{C2} = \frac{1}{2} I_C \omega^2 = \frac{1}{2} K_C^2 m_C \omega^2 = \frac{1}{2} K_C^2 \left(\frac{W_C}{g}\right) \omega^2$



WORK DONE:

BLOCK A: $U_{1-2} = V_{g1} - V_{g2} = W_A y_1 - W_A y_2 = W_A \Delta y$

BLOCK B:



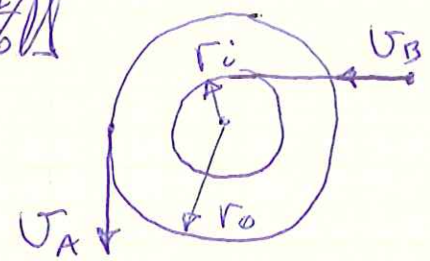
$T_1 = F_F = \mu_K N = \mu_K W_B$

$U_{1-2} = -\mu_K W_B \Delta X_B$

WORK AND ENERGY:

$0 + W_A \Delta y - \mu_K W_B \Delta X_B = \frac{1}{2}\left(\frac{W_A}{g}\right) v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right) v_B^2 + \frac{1}{2} K_C^2 \left(\frac{W_C}{g}\right) \omega^2$

~~$\left[W_A - \mu_K W_B \left(\frac{r_i}{r_o}\right)\right]$~~



$v = r\omega$

$v_A = r_o \omega$

$v_B = r_i \omega$

$$s = r\theta, \quad \Delta X_B = r_i\theta, \quad \Delta y = r_o\theta$$

$$\theta = \frac{\Delta y}{r_o}, \quad \Delta X_B = \left(\frac{r_i}{r_o}\right)\Delta y$$

$$W_A \Delta y - \mu_K W_B\left(\frac{r_i}{r_o}\right)\Delta y = \frac{1}{2g}\left[W_A(r_o\omega)^2 + W_B(r_i\omega)^2 + K_c^2 W_c \omega^2\right]$$

$$\left[W_A - \mu_K W_B\left(\frac{r_i}{r_o}\right)\right]\Delta y = \frac{\omega^2}{2g}\left(W_A r_o^2 + W_B r_i^2 + W_c K_c^2\right)$$

$$\omega = \sqrt{\frac{2g\Delta y\left[W_A - \mu_K W_B\left(\frac{r_i}{r_o}\right)\right]}{\left(W_A r_o^2 + W_B r_i^2 + W_c K_c^2\right)}}$$

$$\omega = \sqrt{\frac{2\left(32.2\,\frac{ft}{s^2}\right)(3\,ft)\left[(25\,LB) - (0.25)(20\,LB)\left(\frac{6}{10}\right)\right]}{\left[(25\,LB)\left(\frac{10}{12}\,ft\right)^2 + (20\,LB)\left(\frac{6}{12}\,ft\right)^2 + (30\,LB)\left(\frac{6.5}{12}\,ft\right)^2\right]}}$$

$$\boxed{\omega = 11.68\ \frac{RAD}{s}}$$

$$v_A = r_o\omega = \left(\frac{10}{12}\,ft\right)\left(11.68\,\frac{RAD}{s}\right) = \boxed{9.732\,\frac{ft}{s}}$$

b) FIND TOTAL $\Delta X_B = \Delta X_{B1} + \Delta X_{B2}$

WHEN BLOCK A HITS THE GROUND,

$$\Delta X_{B1} = \left(\frac{r_i}{r_o}\right)\Delta y = \left(\frac{6}{10}\right)(3\,ft) = 1.8\,ft$$

$$v_B = r_i\omega = \left(\frac{6}{12}\,ft\right)\left(11.68\,\frac{RAD}{s}\right) = 5.84\,\frac{ft}{s}$$

WORK AND ENERGY: $T_2 + U_{2-3} = T_3$

$$T_2 = \frac{1}{2}M_B v_B^2 + \frac{1}{2}I\omega^2, \quad T_3 = 0$$

$$U_{2-3} = -\mu_K W_B \Delta X_{B2}$$

$$\frac{1}{2}\left(\frac{W_B}{g}\right)v_B^2 + \frac{1}{2}k_C^2\left(\frac{W_C}{g}\right)\omega^2 - \mu_K W_B \Delta X_{B2} = 0$$

$$\Delta X_{B2} = \frac{\left(W_B v_B^2 + W_C k_C^2 \omega^2\right)}{2g\mu_K W_B}$$

$$\Delta X_{B2} = \frac{\left[(20^{LB})(5.84\ \tfrac{ft}{s})^2 + (30^{LB})(\tfrac{6.5}{12}\ ft)^2(11.68\ \tfrac{RAD}{s})^2\right]}{2(32.2\ \tfrac{ft}{s^2})(0.25)(20^{LB})}$$

$$\Delta X_{B2} = 5.847^{ft}$$

$$\Delta X_B = 1.8 + 5.847^{ft} = 7.647^{ft}$$