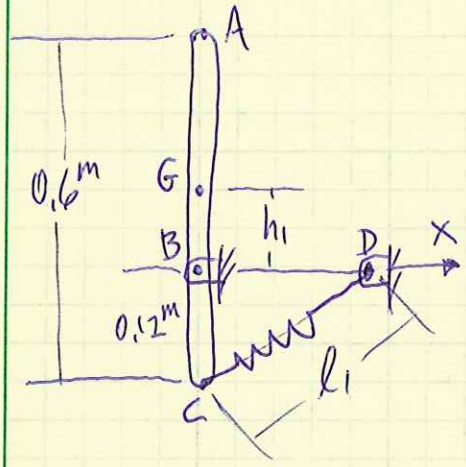
PROB. 17-16

$m = 4\ kg$, $K = 400\ \frac{N}{m}$, $l_u = 0.15\ m$, $\omega_0 = 0$,

FIND ω AT $\theta = 90°$.

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$

POSITION 1



$l_1 = \sqrt{(0.12)^2 + (0.35)^2} = 0.37\ m$

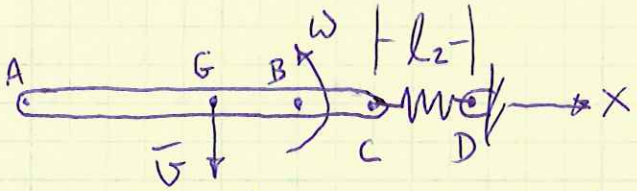
$\Delta l_1 = l_1 - l_u = 0.37 - 0.15 = 0.22\ m$

$h_1 = \frac{1}{2}(0.6) - 0.12 = 0.18\ m$

$T_1 = 0$ SINCE $\omega_0 = 0$

$V_1 = V_e + V_g = \frac{1}{2}K(\Delta l_1)^2 + Wh_1 = \frac{1}{2}K(\Delta l_1)^2 + mgh_1$

POSITION 2



$l_2 = 0.35 - 0.12 = 0.23\ m$

$\Delta l_2 = l_2 - l_u = 0.23 - 0.15$

$\Delta l_2 = 0.08\ m$

$h_2 = 0$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega^2$

$\bar{v}_2 = r\omega = h_1\omega$, $\bar{I} = \frac{1}{12}mL^2$

$T_2 = \frac{1}{2}m(h_1\omega)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2 = \frac{1}{2}m\omega^2\left(h_1^2 + \frac{1}{12}L^2\right)$

$V_2 = V_e + \cancel{V_g}^{\,0} = \frac{1}{2}K(\Delta l_2)^2$

CONSERVATION OF ENERGY:

$$0 + \tfrac{1}{2}K(\Delta l_1)^2 + mgh_1 = \tfrac{1}{2}mw^2\left(h_1^2 + \tfrac{1}{12}L^2\right) + \tfrac{1}{2}K(\Delta l_2)^2$$

$$w = \sqrt{\frac{\left\{\tfrac{1}{2}K\left[(\Delta l_1)^2 - (\Delta l_2)^2\right] + mgh_1\right\}}{\tfrac{1}{2}m\left(h_1^2 + \tfrac{1}{12}L^2\right)}}$$

$$w = \sqrt{\frac{\left\{\tfrac{1}{2}\left(400\,\tfrac{N}{m}\right)\left[(0.22^m)^2 - (0.08^m)^2\right] + \left(4^{kg}\right)\left(9.81\,\tfrac{m}{s^2}\right)\left(0.18^m\right)\right\}}{\tfrac{1}{2}\left(4^{kg}\right)\left[(0.18^m)^2 + \tfrac{1}{12}(0.6^m)^2\right]}}$$

$$\boxed{w = 11.13\ \tfrac{RAD}{s}}$$