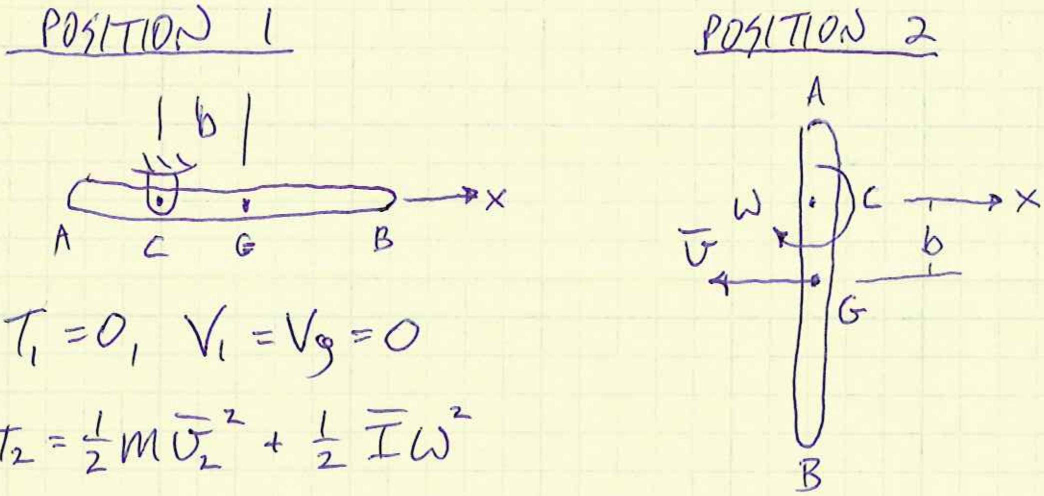
PROB. 17-19

$\omega_0 = 0$

a) FIND b FOR MAXIMUM ω AT $\theta = 90°$

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$



$T_1 = 0, \; V_1 = V_g = 0$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega^2$

$\bar{v}_2 = r\omega = b\omega, \quad \bar{I} = \frac{1}{12} m L^2$

$T_2 = \frac{1}{2} m (b\omega)^2 + \frac{1}{2}\left(\frac{1}{12} m L^2\right)\omega^2 = \frac{1}{2} m \omega^2 \left(b^2 + \frac{1}{12} L^2\right)$

$V_2 = V_g = -mgb$

$0 = \frac{1}{2} m \omega^2 \left(b^2 + \frac{1}{12} L^2\right) - mgb$

$$\omega^2 = \frac{2gb}{\left(b^2 + \frac{1}{12} L^2\right)}$$

THE MAXIMUM ω OCCURS AT

$$\frac{d}{db}\left(\frac{b}{b^2 + \frac{1}{12} L^2}\right) = 0$$

$$\frac{d}{db}\left[b\left(b^2 + \frac{1}{12} L^2\right)^{-1}\right] = 0$$

$$b\left[-\left(b^2 + \frac{1}{12} L^2\right)^{-2}\right](2b) + \left(b^2 + \frac{1}{12} L^2\right)^{-1} = 0$$

$$-2b^2 + \left(b^2 + \tfrac{1}{12}L^2\right) = 0$$

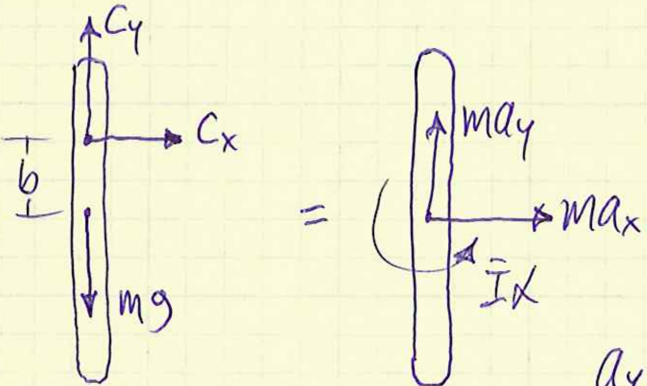
$$b = \sqrt{\frac{L^2}{12}} \boxed{= \frac{L}{\sqrt{12}}}$$

b) FIND CORRESPONDING ω AND REACTION AT C

$$\omega = \sqrt{\frac{2g\frac{L}{\sqrt{12}}}{\frac{1}{12}L^2 + \frac{1}{12}L^2}} = \sqrt{\frac{\frac{2}{\sqrt{12}}\cdot gL}{\frac{1}{6}L^2}} = \sqrt{\frac{\sqrt{12}\cdot g}{L}}$$

$$\boxed{\omega = (12)^{1/4}\sqrt{\frac{g}{L}}}$$

FBD:



$\Sigma F_x = ma_x$:

$C_x = ma_x = ma_t = mr\alpha = mb\alpha$

$\Sigma F_y = ma_y$:

$C_y - mg = ma_y$

$a_y = a_n = r\omega^2 = b\omega^2$

$$a_y = \frac{L}{\sqrt{12}}\cdot\sqrt{12}\cdot\left(\frac{g}{L}\right) = g$$

$$C_y = mg + mg \boxed{= 2mg \uparrow}$$

$\Sigma M_C = \Sigma (M_C)_{EFF}$ $+\circlearrowleft$: $\quad 0 = bma_x + \bar{I}\alpha$

$a_x = a_t = r\alpha = b\alpha$

$bmb\alpha + \bar{I}\alpha = 0, \quad \alpha(b^2m + \bar{I}) = 0$

TRUE ONLY FOR $\alpha = 0 \Rightarrow \boxed{C_x = 0}$