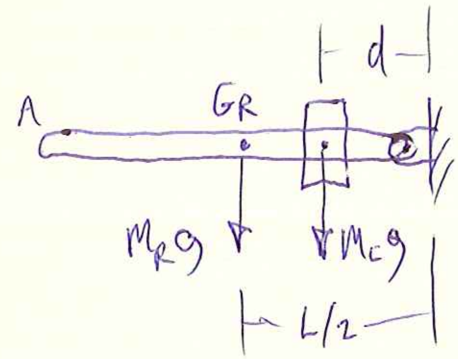
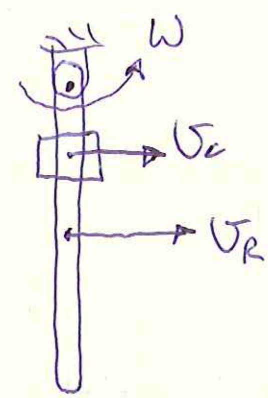
PROB. 17-23

$m_c = 1^{kg}$, $m_R = 3^{kg}$, $L = 0.6^m$, $\omega_0 = 0$, FIND d FOR ω_{max} AT $\theta = 90°$.

POSITION 1



POSITION 2



CONSERVATION OF ENERGY:

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0, \quad V_1 = 0$$

$$T_2 = \frac{1}{2} m_R \bar{v}_R^{\,2} + \frac{1}{2} \bar{I} \omega^2 + \frac{1}{2} m_c \bar{v}_c^{\,2}$$

$$\bar{v}_R = r\omega = \frac{1}{2} L\omega, \quad \bar{I} = \frac{1}{12} m_R L^2, \quad \bar{v}_c = r\omega = d\omega$$

$$T_2 = \frac{1}{2} m_R \left(\frac{1}{2} L\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12} m_R L^2\right)\omega^2 + \frac{1}{2} m_c (d\omega)^2$$

$$T_2 = \frac{\omega^2}{2}\left(\frac{1}{4} m_R L^2 + \frac{1}{12} m_R L^2 + m_c d^2\right)$$

$$T_2 = \frac{\omega^2}{2}\left(\frac{1}{3} m_R L^2 + m_c d^2\right)$$

$$V_2 = V_g = Wh = -m_R g\left(\frac{L}{2}\right) - m_c g(d)$$

$$V_2 = -g\left(\frac{1}{2} m_R L + m_c d\right)$$

$$0 = \frac{\omega^2}{2}\left(\frac{1}{3} m_R L^2 + m_c d^2\right) - g\left(\frac{1}{2} m_R L + m_c d\right)$$

$$\frac{\omega^2}{2g} = \frac{\left(\frac{1}{2}M_R L + M_c d\right)}{\left(\frac{1}{3}M_R L^2 + M_c d^2\right)} = \left(\frac{1}{2}M_R L + M_c d\right)\left(\frac{1}{3}M_R L^2 + M_c d^2\right)^{-1}$$

$$\frac{\omega^2}{2g} = \left[\frac{1}{2}(3^{kg})(0.6^{m}) + (1^{kg})\,d\right]\cdot\left[\frac{1}{3}(3^{kg})(0.6^{m})^2 + (1^{kg})\,d^2\right]^{-1}$$

$$\frac{\omega^2}{2g} = (0.9 + d)(0.36 + d^2)^{-1}$$

TAKE DERIVATIVE WRT d AND SET TO ZERO:

$$(0.9 + d)\left[-(0.36 + d^2)^{-2}(2d)\right] + (0.36 + d^2)^{-1} = 0$$

$$-2d(0.9 + d) + (0.36 + d^2) = 0$$

$$-1.8d - 2d^2 + 0.36 + d^2 = 0$$

$$d^2 + 1.8\,d - 0.36 = 0$$

$$d = \frac{-1.8 \pm \sqrt{(1.8)^2 - 4(-0.36)}}{2}$$

$$d = -0.9 \pm 1.082$$

$$\boxed{d = 0.1817^{m}}$$