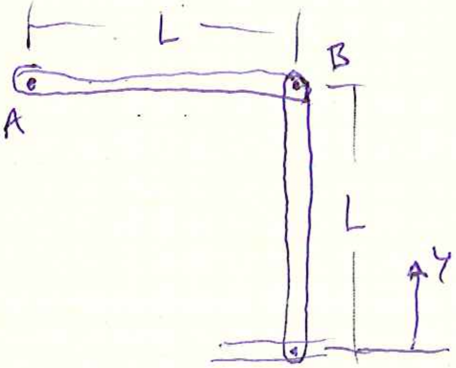
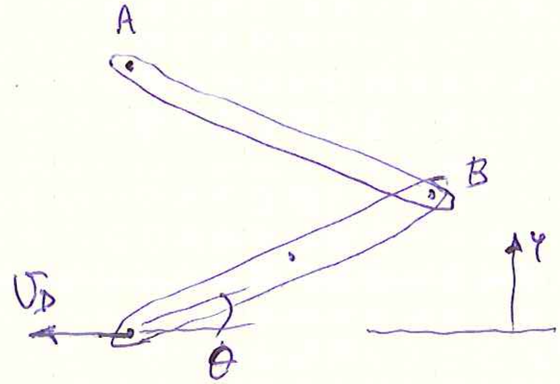
PROB. 17-42

a) FIND v_D

POSITION 1



POSITION 2



$\sin\theta = \frac{L/2}{L}$, $\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$

$T_1 = 0$, $V_1 = (V_g)_1 = W h_{AB} + W h_{BD} = mg\left(L + \frac{1}{2}L\right)$

$V_1 = \frac{3}{2} mgL$

$T_2 = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, BY INSPECTION, $\omega_{AB} = \omega_{BD} = \omega$

FOR AB: $\bar{v} = r\omega = \frac{1}{2}L\omega$, $\bar{I} = \frac{1}{12}mL^2$

FOR BD: $\bar{v} = r\omega = L\cos\theta \cdot \omega$, $\bar{I} = \frac{1}{12}mL^2$

$T_2 = \frac{1}{2}m\left(\frac{1}{2}L\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2 + \frac{1}{2}m\left(L\cos\theta\cdot\omega\right)^2$
$\quad + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2$

$T_2 = mL^2\omega^2\left(\frac{1}{8} + \frac{1}{24} + \frac{1}{2}\cos^2\theta + \frac{1}{24}\right)$

$T_2 = mL^2\omega^2\left(\frac{5}{24} + \frac{1}{2}\cos^2\theta\right)$

TOPS. 35500

PROB. 17-42 CONT.

$V_2 = (V_g)_2 = W h_{AB} + W h_{BD} = mg\left[\left(L - \frac{1}{2}L\sin\theta\right) + \left(\frac{1}{2}L\sin\theta\right)\right]$

$V_2 = mgL$

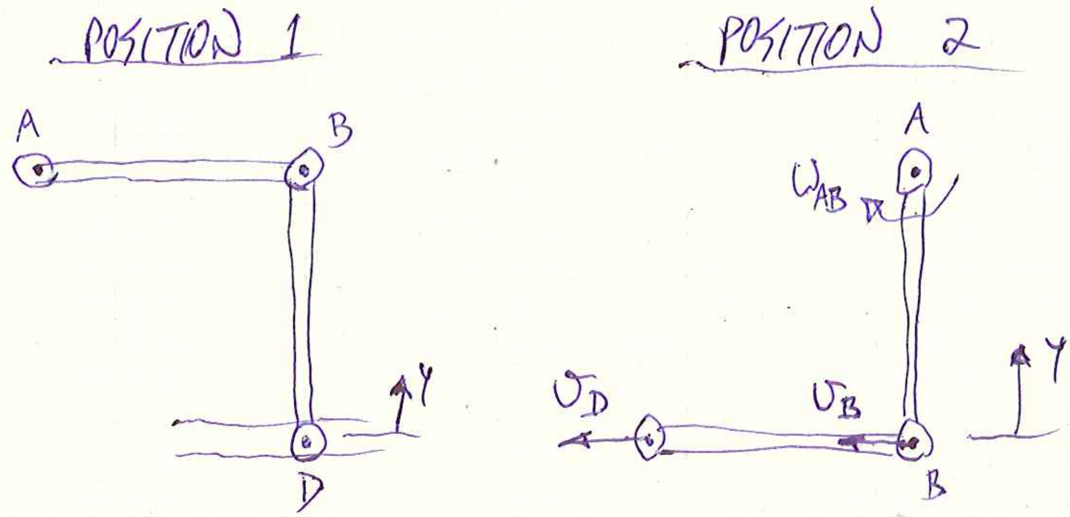
$0 + \frac{3}{2}mgL = mL^2\omega^2\left(\frac{5}{24} + \frac{1}{2}\cos^2\theta\right) + mgL$

$$\omega = \sqrt{\frac{1}{2\left(\frac{5}{24} + \frac{1}{2}\cos^2\theta\right)} \cdot \frac{g}{L}}$$

$$= \sqrt{\frac{1}{2\left(\frac{5}{24} + \frac{1}{2}\cos^2 30^\circ\right)} \cdot \frac{g}{L}} = 0.9258\sqrt{\frac{g}{L}}$$

$v_D = r\omega = L\omega \boxed{= 0.9258\sqrt{gL}} \leftarrow$

b)



BY INSPECTION, $\omega_{BD} = 0$, $v_D = v_B$

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$

$T_1 = 0$, $V_1 = (V_g)_1 = mgL + mg\left(\frac{1}{2}L\right) = \frac{3}{2}mgL$

$T_2 = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$

$T_2 = \frac{1}{2} m \bar{v}_{AB}^2 + \frac{1}{2} \bar{I} \omega_{AB}^2 + \frac{1}{2} m \bar{v}_{BD}^2 + \frac{1}{2} \bar{I} \cancel{\omega_{BD}^2}^{\,0}$

FOR AB: $\bar{v}_{AB} = r\omega_{AB} = \frac{1}{2} L \omega_{AB}$, $\bar{I} = \frac{1}{12} m L^2$

FOR BD: $\bar{v}_{BD} = v_D = v_B = r\omega_{AB} = L\omega_{AB}$

$T_2 = \frac{1}{2} m \left(\frac{1}{2} L \omega_{AB}\right)^2 + \frac{1}{2}\left(\frac{1}{12} m L^2\right)\omega_{AB}^2 + \frac{1}{2} m \left(L\omega_{AB}\right)^2 + 0$

$T_2 = m L^2 \omega_{AB}^2 \left(\frac{1}{8} + \frac{1}{24} + \frac{1}{2}\right) = \frac{2}{3} m L^2 \omega_{AB}^2$

$V_2 = (V_g)_2 = mg\left(\frac{1}{2}L\right) + 0 = \frac{1}{2} mgL$

$0 + \frac{3}{2} mgL = \frac{2}{3} m L^2 \omega_{AB}^2 + \frac{1}{2} mgL$

$\omega_{AB} = \sqrt{\frac{3}{2} \cdot \frac{g}{L}} = 1.225 \sqrt{\frac{g}{L}}$

$v_D = r\omega_{AB} = L\omega_{AB} \boxed{= 1.225\sqrt{gL} \leftarrow}$