

(1)

PROB. 4.115

$W = 75 \text{ lb}$ , ASSUME  $B_x = 0$ , FIND  $T$ ,  $\vec{A}$ ,  $\vec{B}$

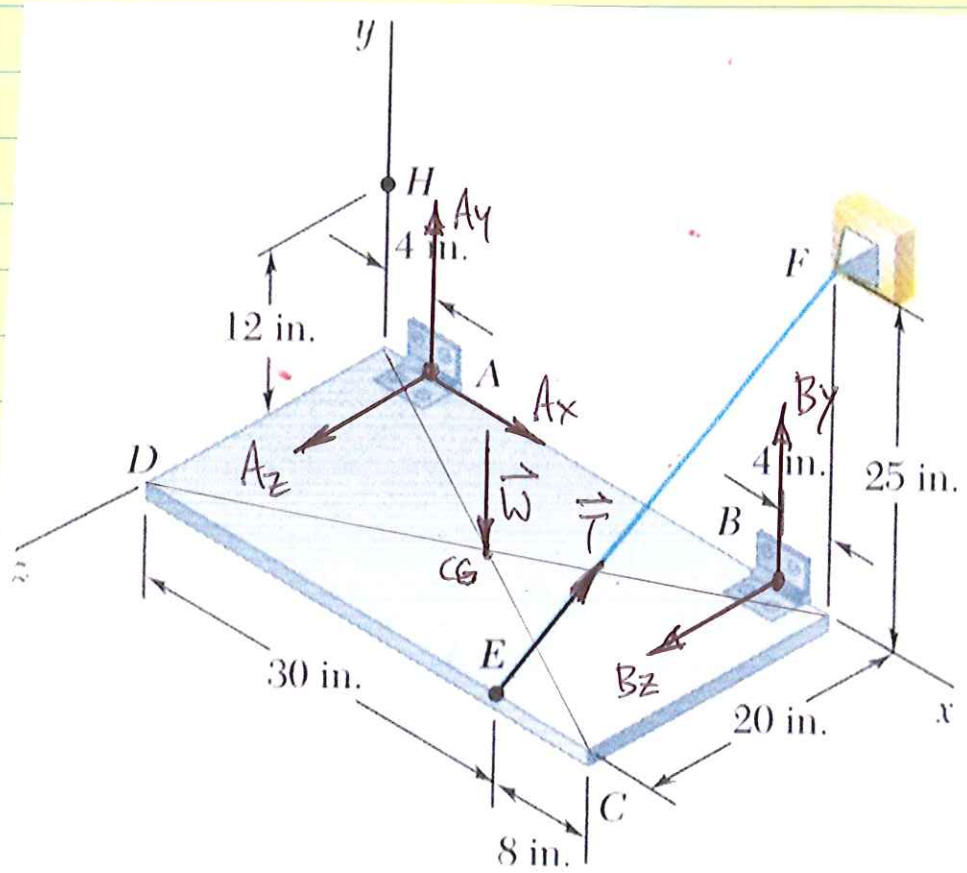


Fig. P4.115

LOCATE POINTS:  $A(4, 0, 0)^{\text{in}}$ ,  $B(34, 0, 0)^{\text{in}}$ ,  
 $E(30, 0, 20)^{\text{in}}$ ,  $F(38, 25, 0)^{\text{in}}$ ,  $CG(19, 0, 10)^{\text{in}}$

DEFINE FORCE VECTORS:

$$\vec{A} = (A_x)\hat{i} + (A_y)\hat{j} + (A_z)\hat{k} \text{ lb}$$

PROB. 4.115 CONT.

(2)

$$\vec{B} = (B_y)\hat{j} + (B_z)\hat{k} \text{ lb}$$

$$\vec{W} = (-75)\hat{j} \text{ lb}$$

$$\vec{T}: dx = x_F - x_E = 38 - 30 = 8 \text{ in}$$

$$dy = y_F - y_E = 25 - 0 = 25 \text{ in}$$

$$dz = z_F - z_E = 0 - 20 = -20 \text{ in}$$

$$d = \sqrt{8^2 + 25^2 + 20^2} = 33 \text{ in}$$

$$T_x = T \frac{dx}{d} = T \left( \frac{8}{33} \right) = 0.242 T$$

$$T_y = T \frac{dy}{d} = T \left( \frac{25}{33} \right) = 0.757 T$$

$$T_z = T \frac{dz}{d} = T \left( \frac{-20}{33} \right) = -0.606 T$$

$$\vec{T} = (0.242 T)\hat{i} + (0.757 T)\hat{j} + (-0.606 T)\hat{k} \text{ lb}$$

SOLVE FOR  $\vec{T}$  DIRECTLY BY SETTING  $\vec{M}_O \cdot \hat{i} = 0$ .

$$\vec{M}_O = \vec{r}_{CG} \times \vec{W} + \vec{r}_E \times \vec{T}$$

$$\vec{r}_{CG} = (19)\hat{i} + (10)\hat{k} \text{ in}, \quad \vec{r}_E = (30)\hat{i} + (20)\hat{k} \text{ in}$$

PROB. 4.115 CONT.

(3)

$$\vec{r}_{CG} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix}$$

$$= [0 - (10)(-75)]\hat{i} - [0]\hat{j} + [(19)(-75) - 0]\hat{k}$$

$$= (750)\hat{i} + (-1425)\hat{k} \text{ N}\cdot\text{m}$$

$$\vec{r}_E \times \vec{T} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 30 & 0 & 20 \\ 0.242T & 0.757T & -0.606T \end{vmatrix}$$

$$= [0 - (20)(0.757T)]\hat{i} - [(30)(-0.606T) - (20)(0.242T)]\hat{j}$$

$$+ [(30)(0.757T) - 0]\hat{k}$$

$$= (-15.1T)\hat{i} + (-23T)\hat{j} + (22.7T)\hat{k} \text{ N}\cdot\text{m}$$

$$\vec{M}_O = (750 - 15.1T)\hat{i} + (-23T)\hat{j}$$

$$+ (-1425 + 22.7T)\hat{k}$$

$$\vec{M}_O \cdot \hat{i} = 750 - 15.1T = 0 \Rightarrow \underline{T = 49.7 \text{ LB}}$$

$$\vec{T} = [0.242(49.7)]\hat{i} + [0.757(49.7)]\hat{j}$$

$$+ [-0.606(49.7)]\hat{k}$$

PROB. 4.115 CONT.

(4)

$$\vec{T} = (12)\hat{i} + (37.6)\hat{j} + (-30.1)\hat{k} \text{ lb}$$

$$\sum F_x = 0: A_x + 12 = 0 \Rightarrow \underline{A_x = -12 \text{ lb}}$$

$$\sum F_y = 0: A_y + B_y + 37.6 - 75 = 0$$

$$A_y = -B_y + 37.4 \quad (1)$$

$$\sum F_z = 0: A_z + B_z - 30.1 = 0$$

$$A_z = -B_z + 30.1 \quad (2)$$

$$\sum \vec{M}_A = \vec{r}_{ACG} \times \vec{W} + \vec{r}_{AE} \times \vec{T} + \vec{r}_{AB} \times \vec{B} = 0 :$$

POSITION VECTORS:

$$\vec{r}_{ACG}: dx = x_{CG} - x_A = 19 - 4 = 15 \text{ in}$$

$$dy = y_{CG} - y_A = 0$$

$$dz = z_{CG} - z_A = 10 - 0 = 10 \text{ in}$$

$$\vec{r}_{ACG} = (15)\hat{i} + (10)\hat{k} \text{ in}$$

$$\vec{r}_{AE}: dx = x_E - x_A = 30 - 4 = 26 \text{ in}$$

$$dy = y_E - y_A = 0$$

$$dz = z_E - z_A = 20 - 0 = 20 \text{ m}$$

$$\vec{r}_{AE} = (26)\hat{i} + (20)\hat{k} \text{ m}$$

$$\vec{r}_{AB}: dx = x_B - x_A = 34 - 4 = 30 \text{ m}, dy = 0, dz = 0$$

$$\vec{r}_{AB} = (30)\hat{i} \text{ m}$$

$$\vec{r}_{AC} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix}$$

$$= [0 - (10)(-75)]\hat{i} - [0]\hat{j} + [(15)(-75) - 0]\hat{k}$$

$$= (750)\hat{i} + (-1125)\hat{k} \text{ m}\cdot\text{N}$$

$$\vec{r}_{AE} \times \vec{T} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 26 & 0 & 20 \\ 12 & 37.6 & -30.1 \end{vmatrix}$$

$$= [0 - (20)(37.6)]\hat{i} - [(26)(-30.1) - (20)(12)]\hat{j}$$

$$+ [(26)(37.6) - 0]\hat{k}$$

PROB. 4.115 CONT.

(6)

$$\vec{r}_{AE} \times \vec{T} = (-752)\hat{i} + (1023)\hat{j} + (978)\hat{k} \text{ N}\cdot\text{m}$$

$$\vec{r}_{AB} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix}$$

$$= [0]\hat{i} - [(30)B_z - 0]\hat{j} + [(30)B_y - 0]\hat{k}$$

$$= (-30B_z)\hat{j} + (30B_y)\hat{k} \text{ N}\cdot\text{m}$$

$$\sum \vec{M}_A = (750 - 752)\hat{i} + (1023 - 30B_z)\hat{j} + (-1125 + 978 + 30B_y)\hat{k} = 0$$

$$B_z = \left(\frac{1023}{30}\right) = 34.1 \text{ LB}$$

$$B_y = \frac{1}{30}(1125 - 978) = 4.9 \text{ LB}$$

$$\textcircled{1}: A_y = -(4.9) + 37.4 = 32.5 \text{ LB}$$

$$\textcircled{2}: A_z = -(34.1) + 30.1 = -4.0 \text{ LB}$$

$$\vec{A} = (-12)\hat{i} + (32.5)\hat{j} + (-4.0)\hat{k} \text{ LB}$$

$$\vec{B} = (4.9)\hat{j} + (34.1)\hat{k} \text{ LB}$$