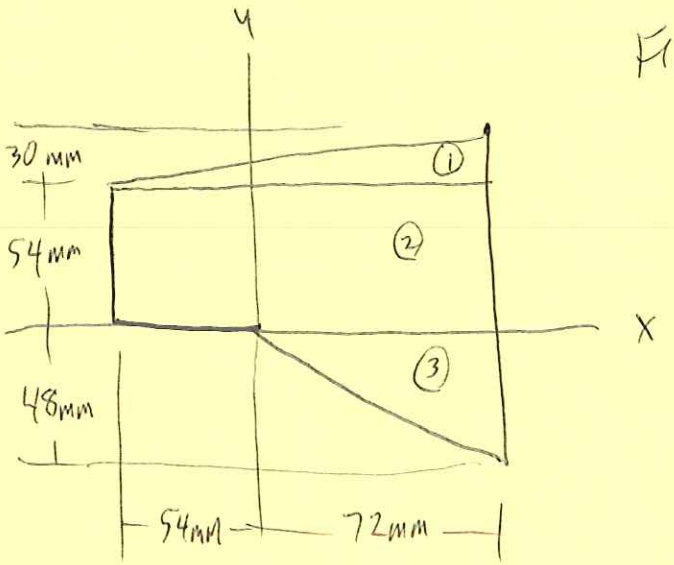
225
P. 215; CENTROID LOCATIONS FOR VARIOUS SHAPES ARE GIVEN. WE CAN USE THESE ELEMENTAL SHAPES TO FIND THE CENTROIDS OF MORE COMPLICATED SHAPES.

EXAMPLE PROB. 5.6

Given: Schematic

FIND CENTROID



SUM MOMENTS:

$$\Sigma M_y: \bar{X}(W_1 + W_2 + W_3) = \bar{X}_1 W_1 + \bar{X}_2 W_2 + \bar{X}_3 W_3$$

$$\Sigma M_x: \bar{Y}(W_1 + W_2 + W_3) = \bar{Y}_1 W_1 + \bar{Y}_2 W_2 + \bar{Y}_3 W_3$$

$\bar{X}$ = X LOCATION OF THE CENTROID OF THE COMPOSITE SHAPE

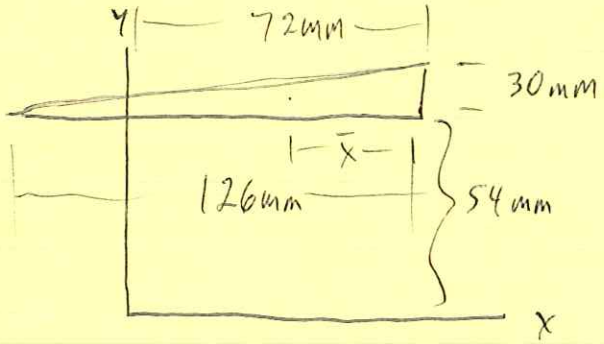
FOR A PLATE OF UNIFORM THICKNESS,

$$Q_y = \bar{X} \cdot \sum_{i=1}^{3} A_i = \sum_{i=1}^{3} \bar{x}_i A_i$$

$$Q_x = \bar{Y} \cdot \sum_{i=1}^{3} A_i = \sum_{i=1}^{3} \bar{y}_i A_i$$

FIND CENTROID OF EACH AREA USING P. 225:

TRIANGLE 1:



$$\bar{y}_1 = \frac{h}{3} = \frac{30}{3} = 10\,mm$$

$$\bar{y}_1 = 54 + 10 = 64\,mm$$

$$\bar{X} = \frac{h}{3} = \frac{126}{3} = 42\,mm$$

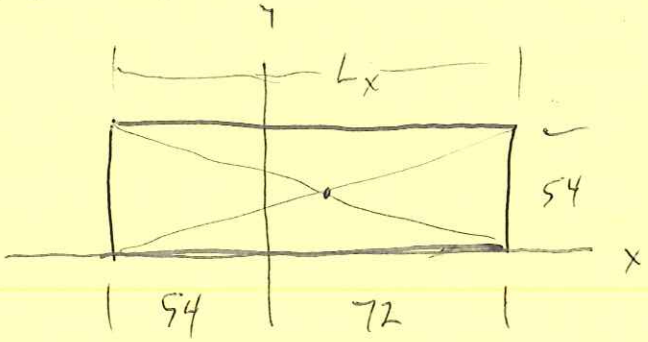
$$\bar{x}_1 = 72 - 42 = 30\,mm$$

$$A_1 = \frac{1}{2}bh = \frac{1}{2}(30\,mm)(126\,mm) = 1890\,mm^2$$

$$\bar{x}_1 A_1 = (30\,mm)(1890\,mm^2) = 5.67 \times 10^4\,mm^3$$

$$\bar{y}_1 A_1 = (64\,mm)(1890\,mm^2) = 1.21 \times 10^5\,mm^3$$

RECTANGLE 2:



$\bar{X}_2 = 54/2 = 27\ mm$

$L_x = 126$, $L_x/2 = 63\ mm$

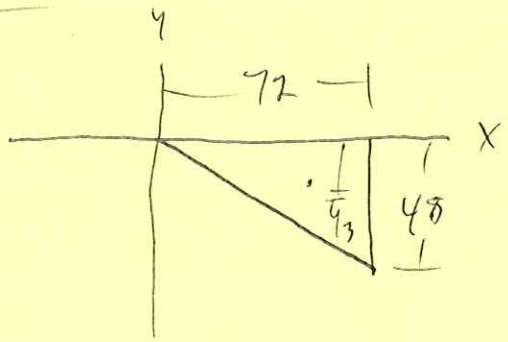
$\bar{X}_2 = 72 - 63 = 9\ mm$

$A_2 = (54)(126) = 6804\ mm^2$

$\bar{X}_2 A_2 = (9)(6804) = 6.12 \times 10^4\ mm^3$

$\bar{Y}_2 A_2 = (27)(6804) = 1.84 \times 10^5\ mm^3$

TRIANGLE 3:



$\bar{Y}_3 = \frac{h}{3} = \frac{-48}{3} = -16\ mm$

$\bar{X} = \frac{72}{3} = 24\ mm$

$\bar{X}_3 = 72 - 24 = 48\ mm$

$A_3 = \frac{1}{2}bh = \frac{1}{2}(48)(72) = 1728\ mm^2$

$\bar{X}_3 A_3 = (48)(1728) = 8.29 \times 10^4\ mm^3$

$\bar{Y}_3 A_3 = (-16)(1728) = -2.76 \times 10^4\ mm^3$

$$\bar{X} = \frac{\Sigma \bar{X}_i A_i}{\Sigma A_i} = \frac{(5.67 \times 10^4) + (6.12 \times 10^4) + (8.29 \times 10^4)}{(1890) + (6804) + (1728)}$$

$\bar{X} = 19.3\ mm$

$$\bar{Y} = \frac{\Sigma \bar{Y}_i A_i}{\Sigma A_i} = \frac{(1.21 \times 10^5) + (1.84 \times 10^5) + (-2.76 \times 10^4)}{(1890) + (6804) + (1728)}$$

$\bar{Y} = 26.6\ mm$

HOMEWORK #5

PROBS. 5.14, 5.23, 5.47, 5.129