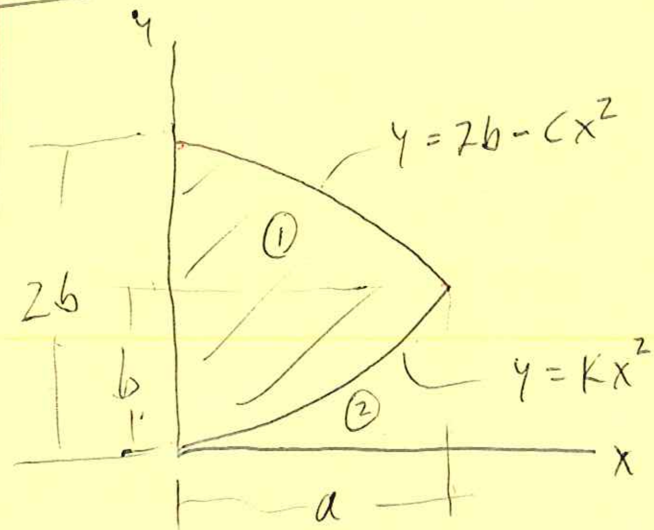
PROB. 5.47



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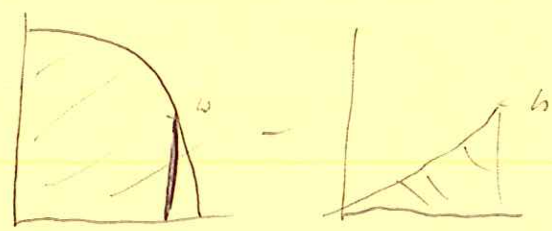

FIND CENTROID USING DIRECT INTEGRATION.

BODY IS SYMMETRIC ABOUT $y = b$

FIND EQUATIONS:

FOR $y = kx^2$, @ $x = a$, $y = b$

$$k = \frac{y}{x^2} = \frac{b}{a^2}$$

$$y = \left(\frac{b}{a^2}\right)x^2$$

FOR $y = 2b - cx^2$, @ $x = a$, $y = b$

$$c = \frac{(2b - y)}{x^2} = \frac{(2b - b)}{a^2} = \frac{b}{a^2}$$

$$y = 2b - \left(\frac{b}{a^2}\right)x^2$$

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$$\bar{X} = \frac{\int \bar{X}_{el}\, dA}{A} \quad , \quad \bar{Y} = \frac{\int \bar{Y}_{EL}\, dA}{A}$$

FOR CURVE 1:

$$\bar{X}_{el} = X \quad , \quad \bar{Y}_{el} = \frac{Y}{2} \quad , \quad dA = Y\,dX$$

$$A = \int dA$$

$$A = \int Y\,dX$$

$$A_1 = \int_0^a (2b - cX^2)\,dX$$

$$= \left[2bX - c\frac{X^3}{3}\right]_0^a$$

$$= \left(2ba - c\frac{a^3}{3}\right)$$

$$= 2ab - \left(\frac{b}{a^2}\right)\left(\frac{a^3}{3}\right)$$

$$= 2ab - \frac{1}{3}ab$$

$$A_1 = \frac{5}{3}ab$$

$$\bar{X}_1 = \frac{1}{A_1}\int \bar{X}_{el}\, dA = \frac{1}{A_1}\int_0^a X(2b - cX^2)\,dX$$

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$$\bar{x}_1 = \frac{1}{A_1}\int_0^a \left(2bx - cx^3\right)dx$$

$$= \frac{1}{A_1}\left[2b\frac{x^2}{2} - c\frac{x^4}{4}\right]_0^a$$

$$= \frac{1}{A_1}\left(a^2 b - c\frac{a^4}{4}\right)$$

$$= \frac{3}{5ab}\left[a^2 b - \left(\frac{b}{a^2}\right)\frac{a^4}{4}\right]$$

$$= \frac{3}{5ab}\left(a^2 b - \frac{a^2 b}{4}\right)$$

$$= \frac{3}{5ab}\left(\frac{3a^2 b}{4}\right)$$

$$\bar{x}_1 = \frac{9}{20}a$$

FOR CURVE 2:

$$A_2 = -\int y\,dx = \int_0^a kx^2 dx$$

$$A_2 = -\left[k\frac{x^3}{3}\right]_0^a$$

$$= -\left(\frac{b}{a^2}\right)\left(\frac{a^3}{3}\right)$$

$$A_2 = -\frac{ab}{3}$$

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$$\bar{X}_2 = \frac{1}{A_2}\int \bar{X}_{el}\, dA$$

$$= \frac{1}{A_2}\int_0^a xy\,dx$$

$$= \frac{3}{ab}\int_0^a x\left(\frac{b}{a^2}x^2\right)dx$$

$$= \frac{3}{a^3}\int_0^a x^3\,dx$$

$$= \frac{3}{a^3}\left[\frac{x^4}{4}\right]_0^a$$

$$= \frac{3}{a^3}\left(\frac{a^4}{4}\right)$$

$$\frac{9\cdot 5}{20\cdot 3} = \frac{3}{4}$$

$$\bar{X}_2 = \frac{3}{4}a$$

$$\bar{X} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i}$$

$$= \frac{\left(\frac{9}{20}a\right)\left(\frac{5}{3}ab\right) + \left(\frac{3}{4}a\right)\left(-\frac{ab}{3}\right)}{\left(\frac{5}{3}ab\right) + \left(-\frac{ab}{3}\right)}$$

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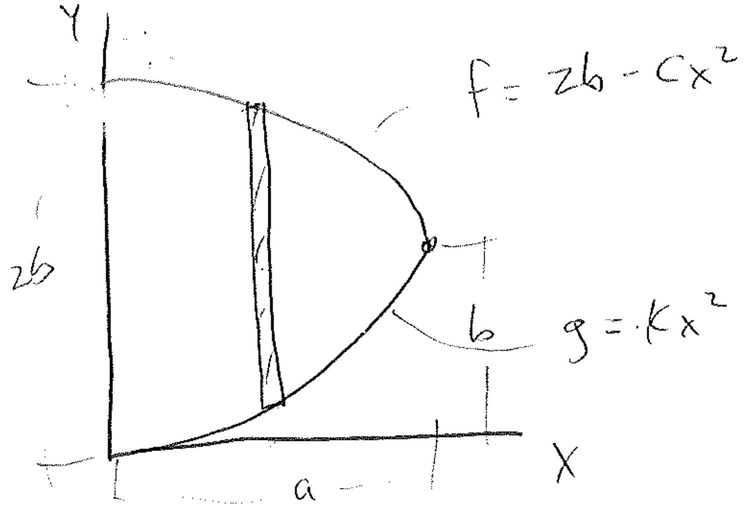
$$= \frac{\frac{3}{4}a^2b - \frac{1}{4}a^2b}{\frac{4}{3}ab}$$

$$\bar{X} = \frac{\frac{1}{2}a^2b}{\frac{4}{3}ab} \qquad \boxed{\bar{X} = \frac{3}{8}a}$$



$$f(x) = 2b - \left(\frac{b}{a^2}\right)x^2$$

$$g(x) = \left(\frac{b}{a^2}\right)x^2$$

$$\bar{X} = \frac{1}{A}\int \bar{X}_{el}\, dA$$

$$\bar{X}_{el} = X, \quad \bar{Y}_{el} = g + \frac{1}{2}(f - g)$$

$$\bar{Y}_{el} = \left(\frac{b}{a^2}\right)x^2 + \frac{1}{2}\left[\left\{2b - \left(\frac{b}{a^2}\right)x^2\right\} - \left(\frac{b}{a^2}\right)x^2\right]$$

$$\bar{Y}_{el} = \left(\frac{b}{a^2}\right)x^2 + \frac{1}{2}\left[2b - 2\left(\frac{b}{a^2}\right)x^2\right]$$

$$\boxed{\bar{Y}_{el} = b}$$

$$dA = \cancel{adx}\,(f - g)\,dx$$

$$dA = \left[\left\{2b - \left(\frac{b}{a^2}\right)x^2\right\} - \left(\frac{b}{a^2}\right)x^2\right]$$

$$\boxed{dA = 2b - 2\left(\frac{b}{a^2}\right)x^2}$$

$$A = \int_0^a (f-g)\,dx$$

$$A = \int_0^a \left[2b - 2\left(\frac{b}{a^2}\right)x^2\right]dx$$

$$A = \left\{2bx - \frac{2}{3}\left(\frac{b}{a^2}\right)x^3\right\}\Big|_0^a$$

$$A = 2ab - \frac{2}{3}\left(\frac{b}{a^2}\right)a^3 \qquad \frac{6}{3} - \frac{2}{3} = \frac{4}{3}$$

$$A = 2ab - \frac{2}{3}ab$$

$$\boxed{A = \frac{4}{3}ab}$$

$$\bar{X} = \frac{3}{4ab}\int_0^a X\cdot\left[2b - 2\left(\frac{b}{a^2}\right)x^2\right]dx$$

$$\bar{X} = \frac{3}{2ab}\int_0^a \left[bx - \left(\frac{b}{a^2}\right)x^3\right]dx$$

$$\bar{X} = \frac{3}{2ab}\left\{\frac{b}{2}x^2 - \frac{1}{4}\left(\frac{b}{a^2}\right)x^4\right\}\Big|_0^a$$

$$\bar{X} = \frac{3}{2ab}\left\{\frac{1}{2}a^2b - \frac{1}{4}a^2b\right\}$$

$$\boxed{\bar{X} = \frac{3}{8}\cdot a}$$