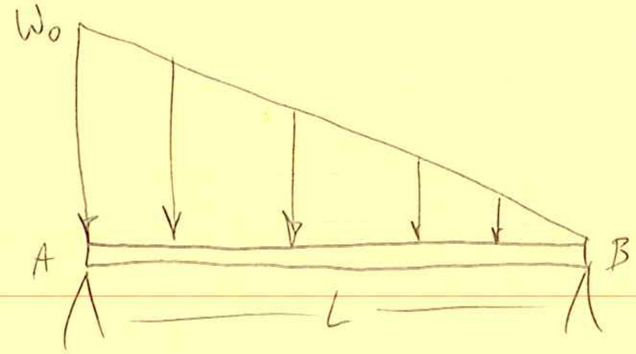
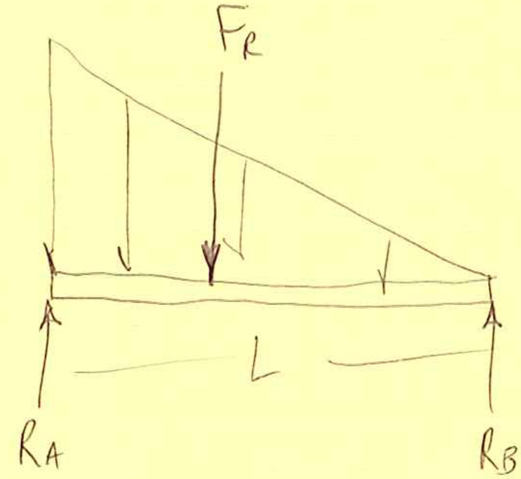
EXAMPLE PROB. 7.85

Given: Loading shown.



FIND A.) EQNS. FOR SHEAR, B-M
B.) mag. & location of max B-M

FBD:



$$F_R = \frac{1}{2} w_0 L$$

$$\sum M_A = 0 \ +\circlearrowleft: \quad -\left(\frac{1}{3}L\right)\left(\frac{1}{2}w_0 L\right) + R_B L = 0$$

$$R_B = \frac{1}{6} w_0 L$$

$$\frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\sum F_y = 0: \quad R_A + R_B - F_R = 0$$

$$R_A = \frac{1}{2} w_0 L - \frac{1}{6} w_0 L = \frac{1}{3} w_0 L$$

$$V_A = R_A = \frac{1}{3} w_0 L \quad , \quad M_A = 0$$

$$w(x) = mx + b$$

$$b = w_0$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{w_2 - w_1}{x_2 - x_1} = \frac{0 - w_0}{L - 0} = -\frac{w_0}{L}$$

$$w(x) = -\frac{w_0}{L} \cdot x + w_0$$

$$V(x) - V_A = -\int_0^x w\,dz$$

$$= -\int_0^x \left(-\frac{w_0}{L} \cdot z + w_0\right) dz$$

$$= -\left[-\frac{w_0}{L} \cdot \frac{z^2}{2} + w_0 z\right]_0^x$$

$$V(x) - V_A = \frac{w_0}{2L} x^2 - w_0 x$$

$$V(x) = w_0\left(\frac{1}{2L} x^2 - x + \frac{1}{3} L\right)$$

BENDING MOMENT:

$$M(x) - M_A = \int_0^x V\,dz$$

$$= \int_0^x \left[w_0\left(\frac{1}{2L} z^2 - z + \frac{1}{3} L\right)\right] dz$$

$$= w_0\left[\frac{1}{2L} \cdot \frac{z^3}{3} - \frac{z^2}{2} + \frac{L}{3} z\right]_0^x$$

$$M(x) - M_A = W_0\left[\frac{1}{6L}x^3 - \frac{1}{2}x^2 + \frac{L}{3}x\right]$$

$$M(x) = W_0\left(\frac{1}{6L}x^3 - \frac{1}{2}x^2 + \frac{L}{3}x\right)$$

FOR MAXIMUM B-M, FIND WHERE $\frac{dM}{dx} = V = 0$

$$\frac{1}{2L}x^2 - x + \frac{1}{3}L = 0$$

$$x = 0.423L$$

SUBSTITUTE INTO $M(x)$:

$$M(0.423L) = 0.0642 W_0 L^2$$

$$x^2 - 2Lx + \frac{2}{3}L^2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2L) \pm \sqrt{4L^2 - 4\cdot\frac{2}{3}L^2}}{2}$$

$$x = \frac{2L \pm 2L\sqrt{1 - \frac{2}{3}}}{2}$$

$$x = L \pm L\sqrt{\tfrac{1}{3}}$$

$x = L\left(1 \pm \sqrt{\tfrac{1}{3}}\right)$ CAN'T BE $1 + \sqrt{\tfrac{1}{3}}$

$$x = L\left(1 - \sqrt{\tfrac{1}{3}}\right)$$