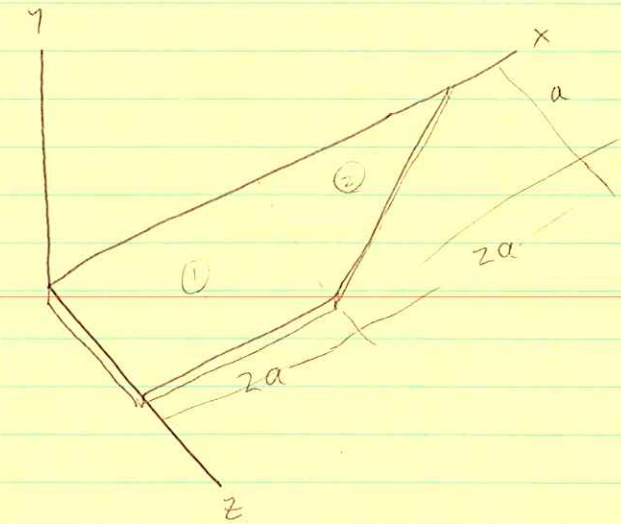
EX.

PROB. 9.117



a) X-AXIS:

$I_x = I_{x,1} + I_{x,2}$

AREA 1: $m_1 = \rho t(a)(2a) = 2\rho t a^2$

$$I_{x,1} = \bar{I}_{x,1} + m_1 d^2$$
$$= \frac{1}{12} m_1 a^2 + m_1 \left(\frac{a}{2}\right)^2$$
$$= (2\rho t a^2)\left[a^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]$$
$$I_{x,1} = \frac{2}{3}\rho t a^4$$

OR

$$I_{x,1} = \rho t \cdot I_{x,1,AREA}$$
$$= \rho t \cdot \frac{1}{3} b h^3$$
$$= \rho t \cdot \frac{1}{3}(2a)(a)^3$$
$$= \frac{2}{3}\rho t a^4$$

AREA 2: $m_2 = \rho t \cdot \frac{1}{2}(2a)(a) = \rho t a^2$

$$I_{x,2} = \rho t \, I_{x,2,AREA}$$
$$= \rho t\left[\frac{1}{12}(2a)(a)^3\right]$$
$$I_{x,2} = \frac{1}{6}\rho t a^4$$

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$I_x = I_{x,1} + I_{x,2}$

$= \frac{2}{3} \rho t a^4 + \frac{1}{6} \rho t a^4$

$I_x = \frac{5}{6} \rho t a^4$

$m = m_1 + m_2$

$= 2 \rho t a^2 + \rho t a^2$

$m = 3 \rho t a^2$

$I_x = (3 \rho t a^2)(\frac{1}{3} \cdot \frac{5}{6} a^2)$

$\boxed{I_x = \frac{5}{18} m a^2}$ IN TERMS OF MASS

(b) Y-AXIS:

$I_y = I_x + I_z$

$I_z = I_{z,1} + I_{z,2}$

AREA 1: $m_1 = 2 \rho t a^2$

$I_{z,1} = \bar{I}_{z,1} + m_1 d^2$

$= \frac{1}{12} m_1 (2a)^2 + m_1 (a)^2 = (2\rho t a^2)(\frac{4a^2}{12} + a^2)$

$= (2 \rho t a^2)(\frac{1}{3} + 1)$

$I_{z,1} = \frac{8}{3} \rho t a^4$

OR $I_{z,1} = \rho t \cdot I_{z,1,\text{AREA}}$

$= \rho t \cdot \frac{1}{3} b h^3$

$= \rho t \cdot \frac{1}{3} (a)(2a)^3$

$= \frac{8}{3} \rho t a^4$

AREA 2: $m_2 = \rho t a^2$

9.117 CONT.

$$I_{z,2} = \rho t \cdot I_{z,2,AREA}$$

$$= \rho t\left(\bar{I}_{z,2} + Ad^2\right)$$

$$= \rho t\left\{\frac{1}{36}(a)(2a)^3 + \frac{1}{2}(a)(2a)\cdot\left[(2a) + \frac{1}{3}(2a)\right]^2\right\}$$

$$I_{z,2} = \frac{22}{3}\rho t a^4$$

$$I_z = I_{z,1} + I_{z,2}$$

$$= \frac{8}{3}\rho t a^4 + \frac{22}{3}\rho t a^4$$

$$I_z = 10\rho t a^4$$

$$m = 3\rho t a^2$$

$$I_z = (3\rho t a^2)\left(\frac{1}{3}\cdot 10a^2\right)$$

$$I_z = \frac{10}{3}ma^2$$

$$I_y = I_x + I_z$$

$$= \frac{5}{18}ma^2 + \frac{10}{3}ma^2$$

$$\boxed{I_y = \frac{65}{18}ma^2}$$