Tenth Edition

Vector Mechanics for Engineers: Dynamics

ME 2210: DYNAMICS

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Course Homepage: https://web1.cs.wright.edu/people/faculty/sthomas/dynamics.html

Class Hours: T Th 2:00 p.m. to 3:20 p.m.

Office Hours: T Th 1:00 p.m. to 1:55 p.m., 3:25 p.m. to 4:55 p.m., or by appointment

Text: *Vector Mechanics for Engineers: Dynamics,* Beer, Johnston, and Cornwell, McGraw-Hill

Course Delivery: The lectures will be delivered using Webex. It is expected that students will be present and participate during Class Hours. Bonus points will be awarded for participation during Class Hours.

Meeting Location: wright.webex.com
Meeting Number: 731 847 872
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Vector Mechanics for Engineers: Dynamics

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Important topics from Statics that are needed to study Dynamics:

Vector manipulation: Addition, cross product, dot product
Vector representation of forces, moments
Free-body diagrams
Friction
Mass moments of inertia

Important topics to be learned in Dynamics:

- Newton's 2nd Law: $\sum \vec{F} = m\vec{a}, \sum \vec{M} = I\vec{\alpha}$ (I = Mass Moment of Inertia, $\vec{\alpha}$ = Angular Acceleration)
- Kinematics: Relates time, displacement, velocity and acceleration without considering the forces or moments causing the motion
- Kinetics: Relates forces, moments, mass of the body and shape of the body to predict the motion of the body
- Linear momentum and angular momentum: $\sum \vec{F} = \dot{\vec{L}}, \quad \sum \vec{M} = \dot{\vec{H}}$

($\dot{\vec{L}}$ = Rate of Change of Linear Momentum,
$\dot{\vec{H}}$ = Rate of Change of Angular Momentum)

- Kinetic energy and work done: $U_{1-2} = T_2 - T_1$ (U = Work Done, T = Kinetic Energy)
- Principle of conservation of energy: $T_1 + V_1 = T_2 + V_2$ (V = Potential Energy)

Chapter 11: Kinematics Of Particles

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CHAPTER

11

VECTOR MECHANICS FOR ENGINEERS:

DYNAMICS

Ferdinand P. Beer
E. Russell Johnston, Jr.
Phillip J. Cornwell
Lecture Notes:
Brian P. Self
California Polytechnic State University

Kinematics of Particles

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Vector Mechanics for Engineers: Dynamics

Contents

Introduction
Rectilinear Motion: Position, Velocity & Acceleration
Determination of the Motion of a Particle
Uniform Rectilinear-Motion
Uniformly Accelerated Rectilinear-Motion
Motion of Several Particles: Relative Motion
Motion of Several Particles: Dependent Motion
Curvilinear Motion: Position, Velocity & Acceleration
Rectangular Components of Velocity and Acceleration
Motion Relative to a Frame in Translation
Tangential and Normal Components
Radial and Transverse Components

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Introduction

- **Dynamics includes:**

Kinematics: study of the geometry of motion.
Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion.

Kinetics: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

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Introduction

- **Particle kinematics includes:**

  - ***Rectilinear motion***: position, velocity, and acceleration of a particle as it moves along a straight line.

  - ***Curvilinear motion***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

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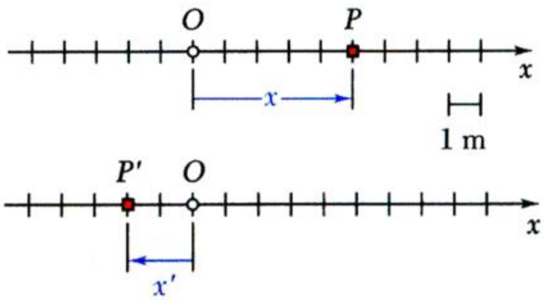
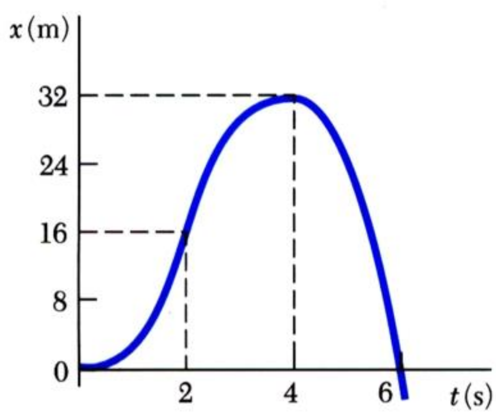
Rectilinear Motion: Position, Velocity & Acceleration



- ***Rectilinear motion:*** particle moving along a straight line
- ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.



- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time t.
- May be expressed in the form of a function, e.g., $x = 6t^2 - t^3$

  or in the form of a graph x vs. t.

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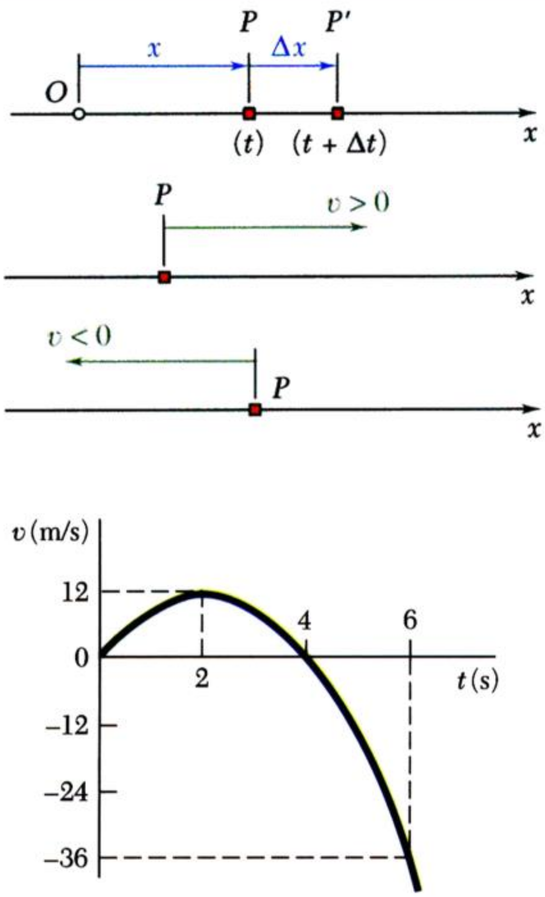
Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle which occupies position P at time t and P' at $t + \Delta t$,

$$\textit{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\textit{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity (a vector) may be positive or negative. Magnitude of velocity is referred to as *particle speed* (a scalar).

- From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

e.g., $x = 6t^2 - t^3$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

11 - 7

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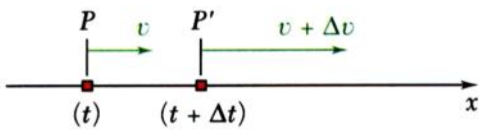
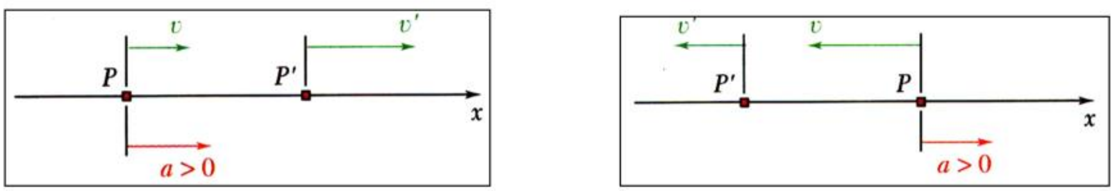
Rectilinear Motion: Position, Velocity & Acceleration



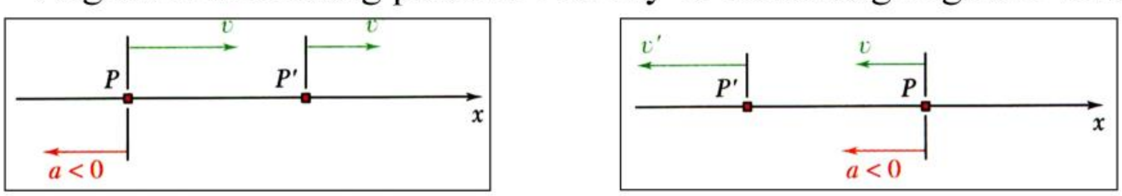
- Consider particle with velocity v at time t and v' at $t + \Delta t$,

$$\textit{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- Instantaneous acceleration (a vector) may be:
  - Positive: increasing positive velocity or decreasing negative velocity



  - Negative: decreasing positive velocity or increasing negative velocity



CHAPTER 11: Rectilinear Motion: Position, Velocity & Acceleration

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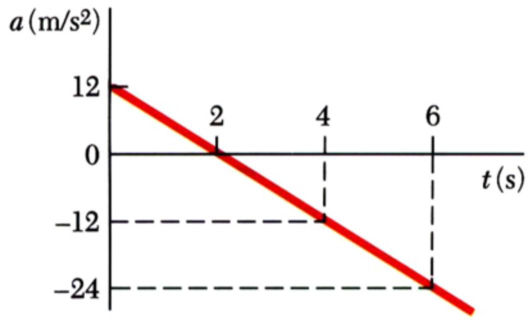
Rectilinear Motion: Position, Velocity & Acceleration



- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

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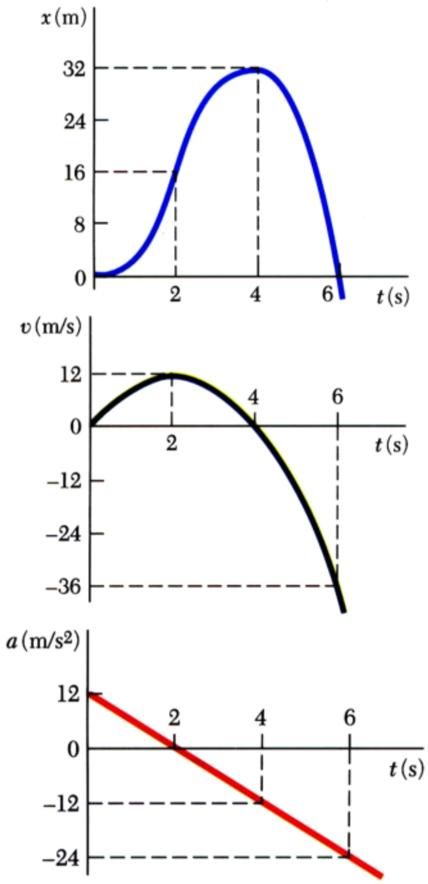
Rectilinear Motion: Position, Velocity & Acceleration



- From our example,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- What are x, v, and a at $t = 2$ s ?
- What are x, v, and a at $t = 4$ s ?

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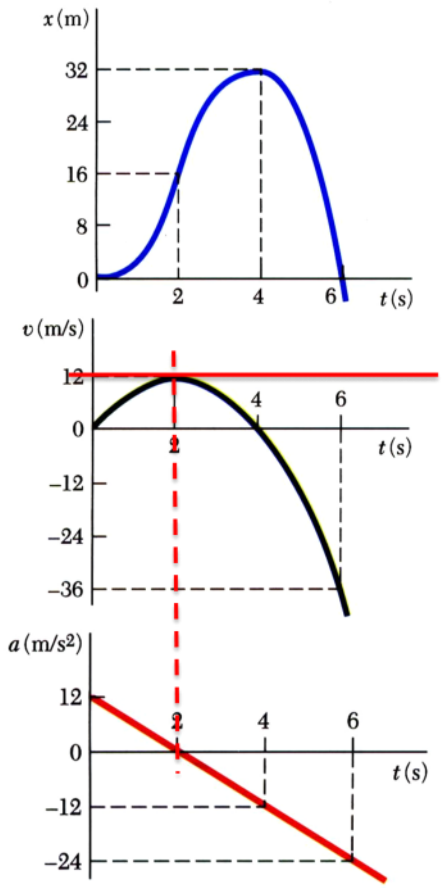
Rectilinear Motion: Position, Velocity & Acceleration



- From our example,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- What are x, v, and a at $t = 2$ s ?

  - at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$

- Note that v_{max} occurs when $a = 0$, and that the slope of the velocity curve is zero at this point.

- What are x, v, and a at $t = 4$ s ?

  - at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2

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Determination of the Motion of a Particle

- **We often determine accelerations from the forces applied (kinetics will be covered later)**
- **Generally there are three classes of motion**
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = f(x)$
  - acceleration given as a function of *velocity*, $a = f(v)$

Force is a function of position: A spring

Force is a function of velocity: Aerodynamic drag

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Acceleration as a Function of Time, Position, or Velocity

If….	Kinematic relationship	Integrate
$a = a(t)$	$\frac{dv}{dt} = a(t)$	$\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\,dt$
$a = a(x)$	$dt = \frac{dx}{v}$ and $a = \frac{dv}{dt}$ → $v\,dv = a(x)\,dx$	$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$
$a = a(v)$	$\frac{dv}{dt} = a(v)$	$\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$
	$v\frac{dv}{dx} = a(v)$	$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{a(v)}$

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11 - 13

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Uniform Rectilinear Motion

During free-fall, a parachutist reaches terminal velocity when her weight equals the drag force. If motion is in a straight line, this is uniform rectilinear motion.

For a particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant.

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

Careful – these only apply to uniform rectilinear motion!

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Uniformly Accelerated Rectilinear Motion

If forces applied to a body are constant (and in a constant direction), then you have uniformly accelerated rectilinear motion.

Another example is free-fall when drag is negligible

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Uniformly Accelerated Rectilinear Motion

For a particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant. You may recognize these constant acceleration equations from your physics courses.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_0^t dt \qquad v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_0^t \left(v_0 + at\right) dt \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Careful – these only apply to uniformly accelerated rectilinear motion!

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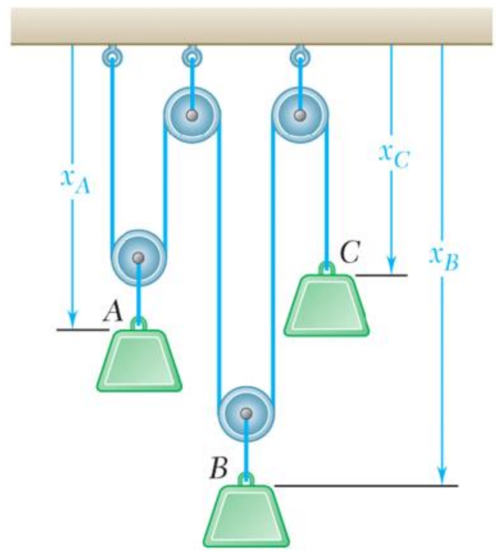
Motion of Several Particles

We may be interested in the motion of several different particles, whose motion may be independent or linked together.



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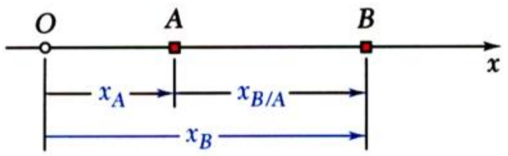
Motion of Several Particles: Relative Motion



- For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.

$$x_{B/A} = x_B - x_A = \text{ relative position of } B \text{ with respect to } A$$

$$x_B = x_A + x_{B/A}$$

$$v_{B/A} = v_B - v_A = \text{ relative velocity of } B \text{ with respect to } A$$

$$v_B = v_A + v_{B/A}$$

$$a_{B/A} = a_B - a_A = \text{ relative acceleration of } B \text{ with respect to } A$$

$$a_B = a_A + a_{B/A}$$

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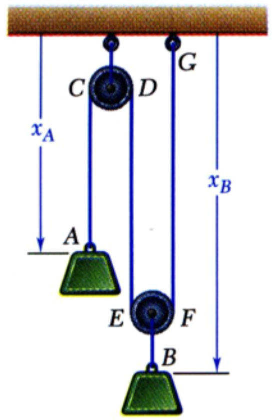
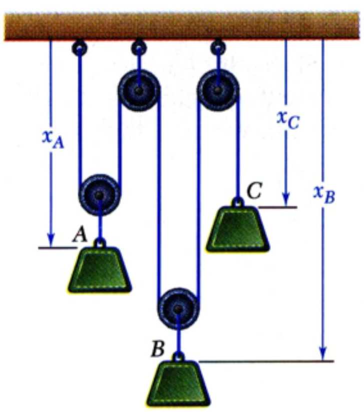
Motion of Several Particles: Dependent Motion





- Position of a particle may *depend* on position of one or more other particles.
- Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.

$$x_A + 2x_B = \text{constant} \quad \text{(one degree of freedom)}$$

- Positions of three blocks are dependent.

$$2x_A + 2x_B + x_C = \text{constant (two degrees of freedom)}$$

- For linearly related positions, similar relations hold between velocities and accelerations.

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

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Curvilinear Motion: Position, Velocity & Acceleration

The softball and the car both undergo curvilinear motion.

- A particle moving along a curve other than a straight line is in *curvilinear motion*.

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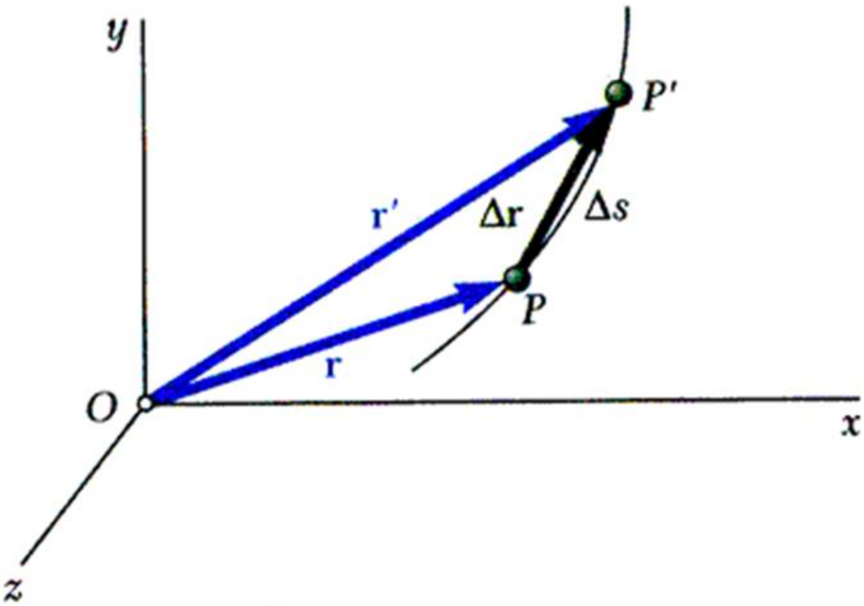
Curvilinear Motion: Position, Velocity & Acceleration

- The *position vector* of a particle at time *t* is defined by a vector between origin *O* of a fixed reference frame and the position occupied by particle.

- Consider a particle which occupies position *P* defined by $\vec{r}$ at time *t* and *P'* defined by $\vec{r}'$ at $t + \Delta t$,



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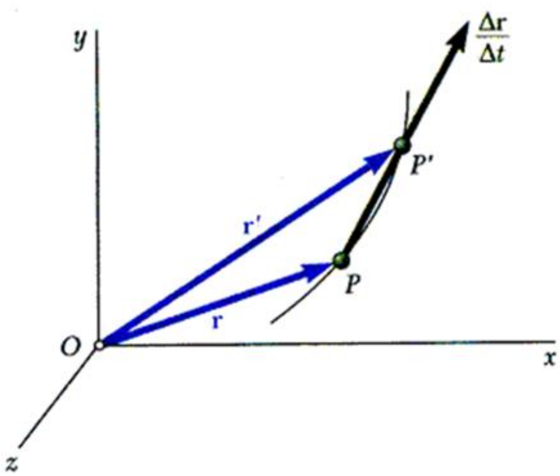
Curvilinear Motion: Position, Velocity & Acceleration

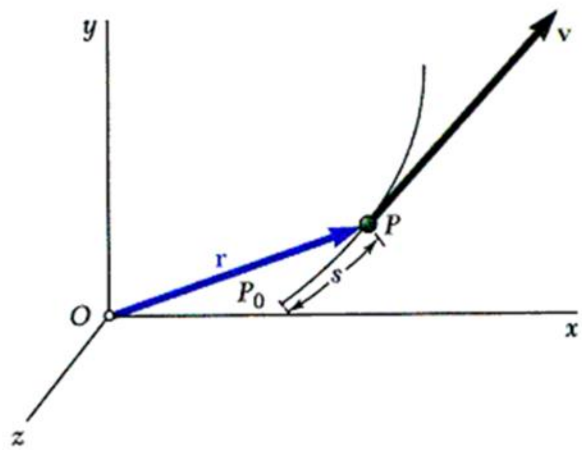
Instantaneous velocity (vector)

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



Instantaneous speed (scalar)

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$



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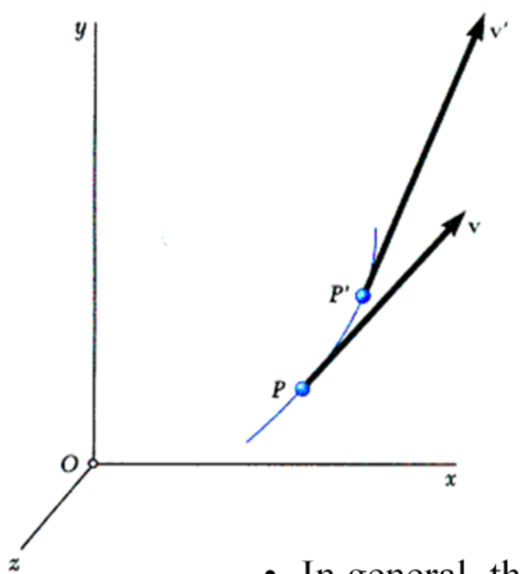
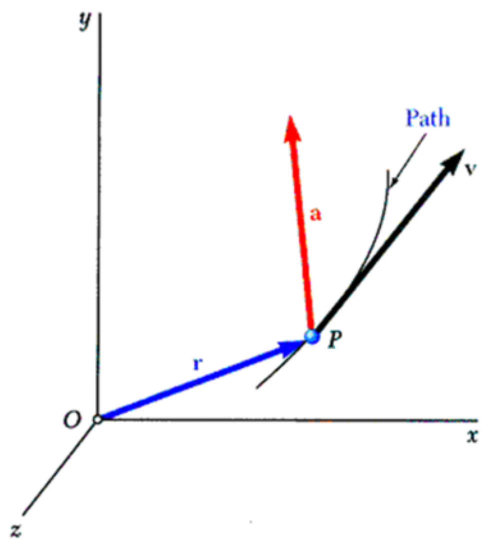
Curvilinear Motion: Position, Velocity & Acceleration

- Consider velocity $\vec{v}$ of a particle at time t and velocity $\vec{v}'$ at $t + \Delta t$,

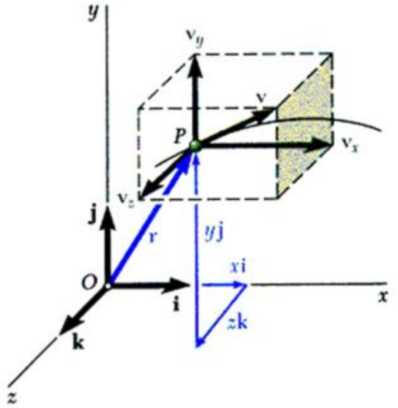
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \text{ instantaneous acceleration (vector)}$$

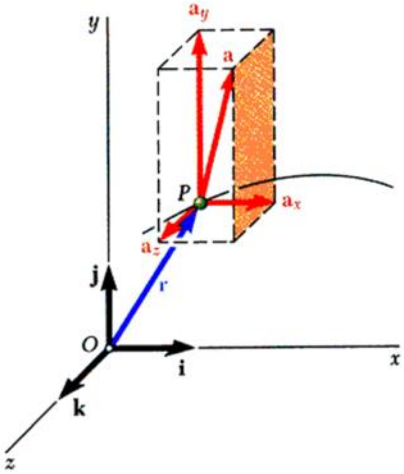




- In general, the acceleration vector is not tangent to the particle path like the velocity vector.

Rectangular Components of Velocity & Acceleration





- When position vector of particle *P* is given by its rectangular components,

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

- Velocity vector,

$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

$$= v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

- Acceleration vector,

$$\vec{a} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} + \frac{d^2z}{dt^2}\vec{k} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$

$$= a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

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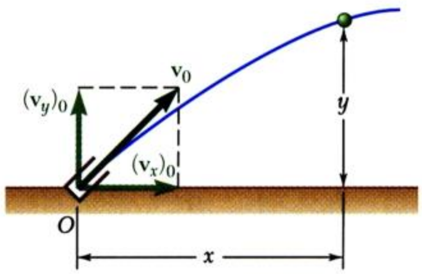
Rectangular Components of Velocity & Acceleration



- Rectangular components are particularly effective when component accelerations can be integrated independently, e.g., motion of a projectile,

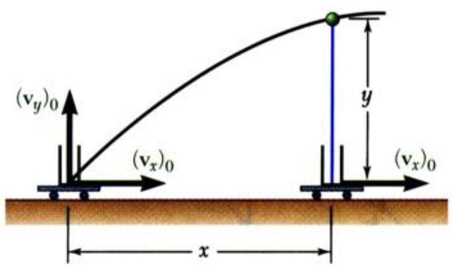
$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

with initial conditions,

$$x_0 = y_0 = z_0 = 0 \qquad (v_x)_0, (v_y)_0, (v_z)_0 = 0$$

Integrating twice yields

$$v_x = (v_x)_0 \quad v_y = (v_y)_0 - gt \qquad v_z = 0$$
$$x = (v_x)_0 t \quad y = (v_y)_0 y - \tfrac{1}{2} g t^2 \quad z = 0$$



- Motion in horizontal direction is uniform.
- Motion in vertical direction is uniformly accelerated.
- Motion of projectile could be replaced by two independent rectilinear motions.

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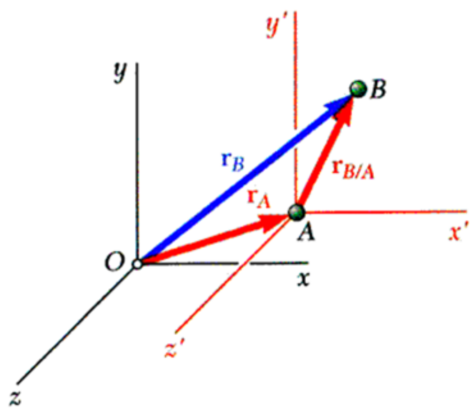
Motion Relative to a Frame in Translation



- Designate one frame as the *fixed frame of reference*. All other frames not rigidly attached to the fixed reference frame are *moving frames of reference*.
- Position vectors for particles *A* and *B* with respect to the fixed frame of reference *Oxyz* are $\vec{r}_A$ and $\vec{r}_B$.
- Vector $\vec{r}_{B/A}$ joining *A* and *B* defines the position of *B* with respect to the moving frame *Ax'y'z'* and

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Differentiating twice,

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{v}_{B/A} = \text{velocity of } B \text{ relative to } A.$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \vec{a}_{B/A} = \text{acceleration of } B \text{ relative to } A.$$

- Absolute motion of *B* can be obtained by combining motion of *A* with relative motion of *B* with respect to moving reference frame attached to *A*.

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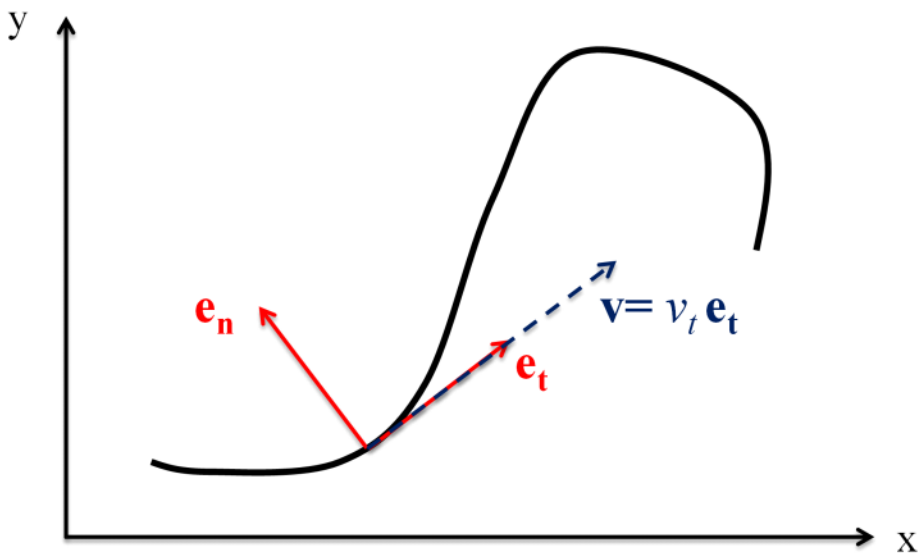
Tangential and Normal Components



ρ = the instantaneous radius of curvature

$$\mathbf{v} = v\,\mathbf{e}_t$$

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n$$

- The tangential direction ($\mathbf{e}_t$) is tangent to the path of the particle. This velocity vector of a particle is in this direction
- The normal direction ($\mathbf{e}_n$) is perpendicular to $\mathbf{e}_t$ and points towards the inside of the curve.
- The acceleration can have components in both the $\mathbf{e}_n$ and $\mathbf{e}_t$ directions

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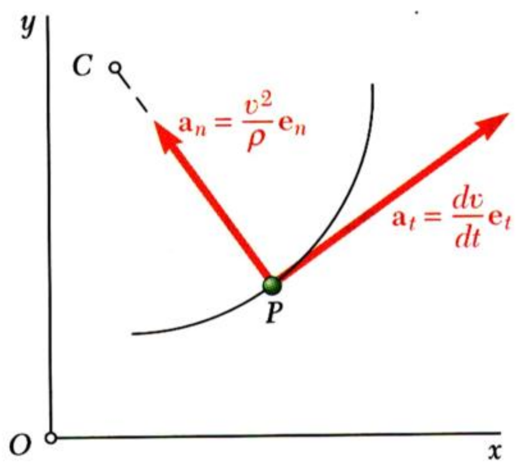
Tangential and Normal Components



$$\vec{v} = v\vec{e}_t$$

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n \qquad a_t = \frac{dv}{dt} \quad a_n = \frac{v^2}{\rho}$$

- The tangential component of acceleration reflects change of speed and the normal component reflects change of direction.
- The tangential component may be positive or negative. Normal component always points toward center of path curvature.

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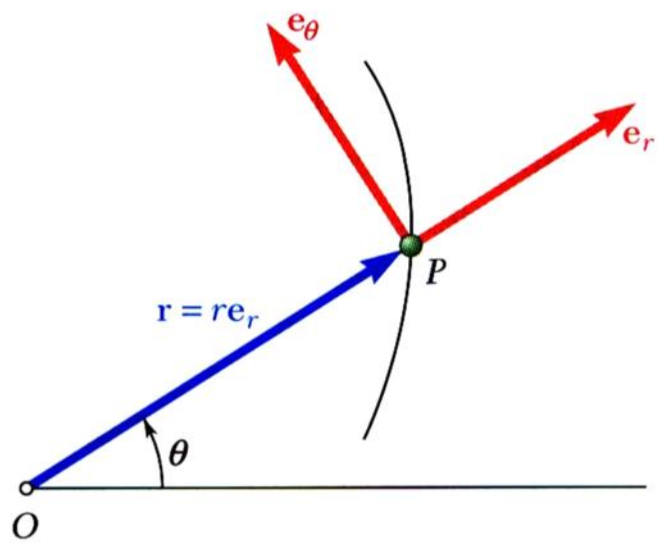
Radial and Transverse Components

- The position of a particle P is expressed as a distance r from the origin O to P – this defines the radial direction $\mathbf{e_r}$. The transverse direction $\mathbf{e_\theta}$ is perpendicular to $\mathbf{e_r}$

$$\vec{r} = r\vec{e}_r$$



- The particle velocity vector is

$$\vec{v} = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$$

- The particle acceleration vector is

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

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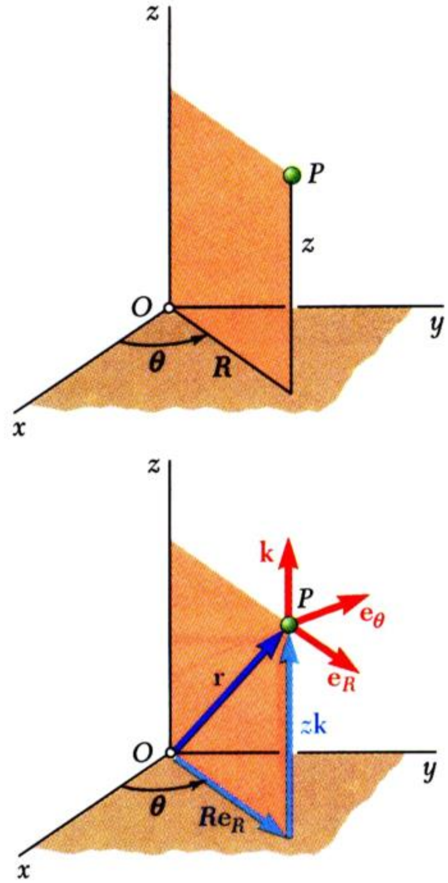
Vector Mechanics for Engineers: Dynamics

Radial and Transverse Components



- When particle position is given in cylindrical coordinates, it is convenient to express the velocity and acceleration vectors using the unit vectors $\vec{e}_R, \vec{e}_\theta$, and $\vec{k}$.

- Position vector,

$$\vec{r} = R\vec{e}_R + z\vec{k}$$

- Velocity vector,

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta + \dot{z}\vec{k}$$

- Acceleration vector,

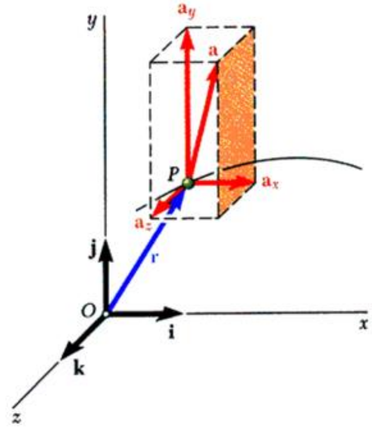
$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{R} - R\dot{\theta}^2\right)\vec{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\vec{e}_\theta + \ddot{z}\vec{k}$$

Tenth Edition

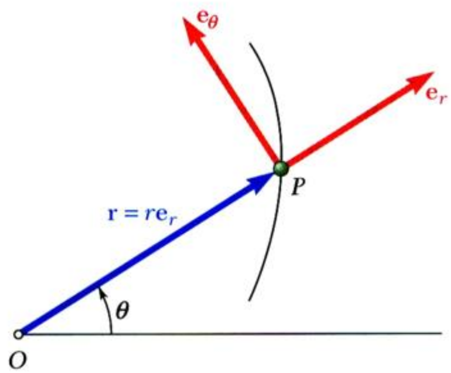
Vector Mechanics for Engineers: Dynamics

Summary of Useful Coordinate Systems

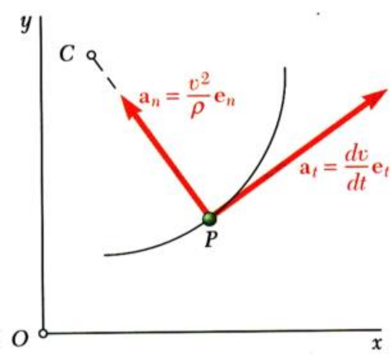
- Rectangular Coordinates: $\hat{\imath}, \hat{\jmath}, \hat{k}$



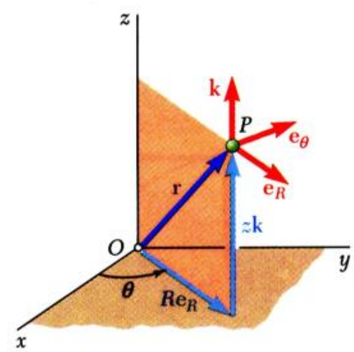
- Radial and Transverse Coordinates: $\vec{e}_r, \vec{e}_\theta$



- Tangential and Normal Coordinates: $\vec{e}_t, \vec{e}_n$



- Cylindrical Coordinates: $\vec{e}_R, \vec{e}_\theta$, and $\vec{k}$.



Mc Graw Hill

11 - 32

Problem 11.15

11.15 The acceleration of a particle is defined by the relation $a = -k/x$. It has been experimentally determined that $v = 15$ ft/s when $x = 0.6$ ft and that $v = 9$ ft/s when $x = 1.2$ ft. Determine (*a*) the velocity of the particle when $x = 1.5$ ft, (*b*) the position of the particle at which its velocity is zero.

Problem 11.24

11.24 A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 25 ft/s. Assuming the ball experiences a downward acceleration of $a = 10 - 0.9v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

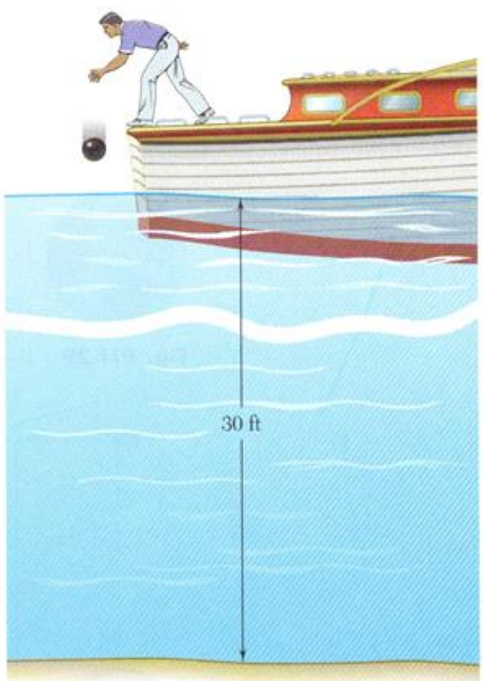


Fig. P11.24

Problem 11.41

11.41 Automobiles A and B are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 1.8 ft/s^2 and that B has a constant deceleration of 1.2 ft/s^2, determine (a) when and where A will overtake B, (b) the speed of each automobile at that time.

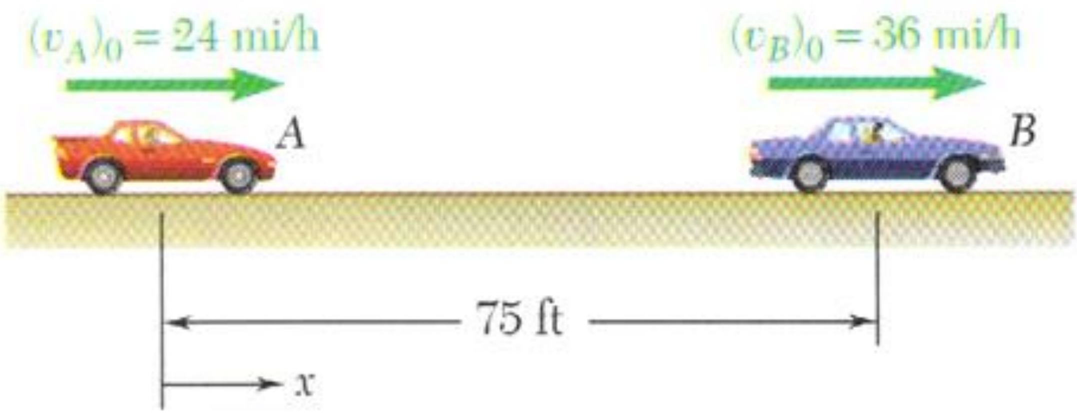


Fig. P11.41

Problem 11.47

11.47 Slider block A moves to the left with a constant velocity of 6 m/s. Determine (*a*) the velocity of block B, (*b*) the velocity of portion D of the cable, (*c*) the relative velocity of portion C of the cable with respect to portion D.

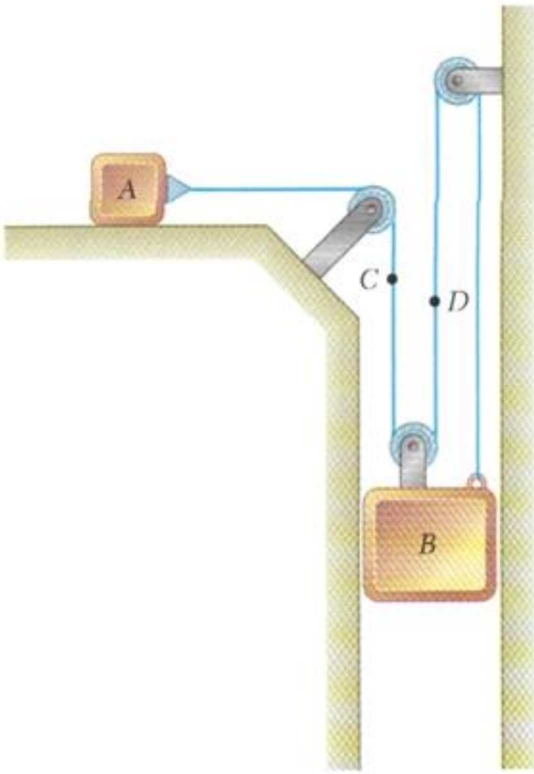


Fig. P11.47 and P11.48

Problem 11.100

11.100 A baseball pitching machine "throws" baseballs with a horizontal velocity $\mathbf{v}_0$. Knowing that height h varies between 31 in. and 42 in., determine (*a*) the range of values of v_0, (*b*) the values of α corresponding to h = 31 in. and h = 42 in.

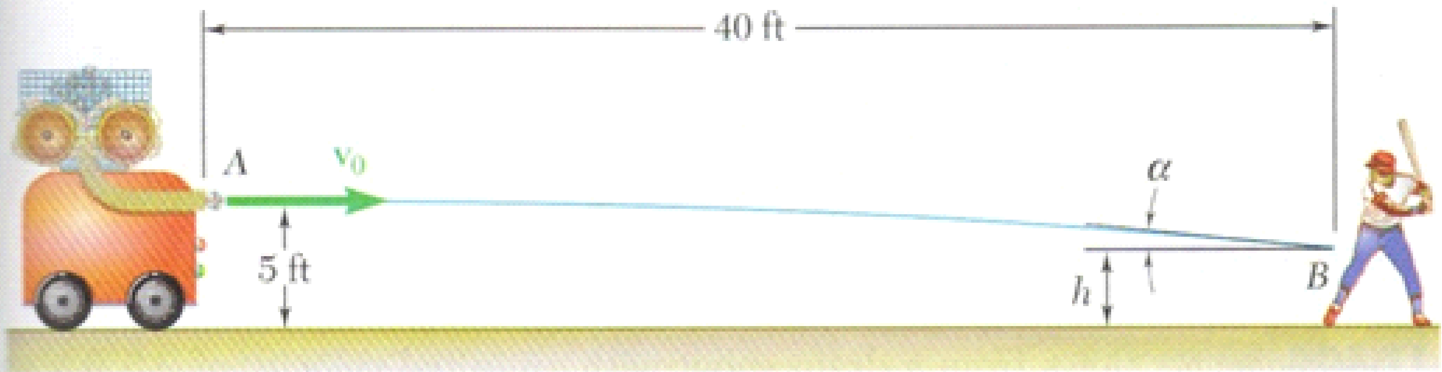


Fig. P11.100

Problem 11.123

11.123 Knowing that at the instant shown block *A* has a velocity of 8 in./s and an acceleration of 6 in./s^2 both directed down the incline, determine (*a*) the velocity of block *B*, (*b*) the acceleration of block *B*.

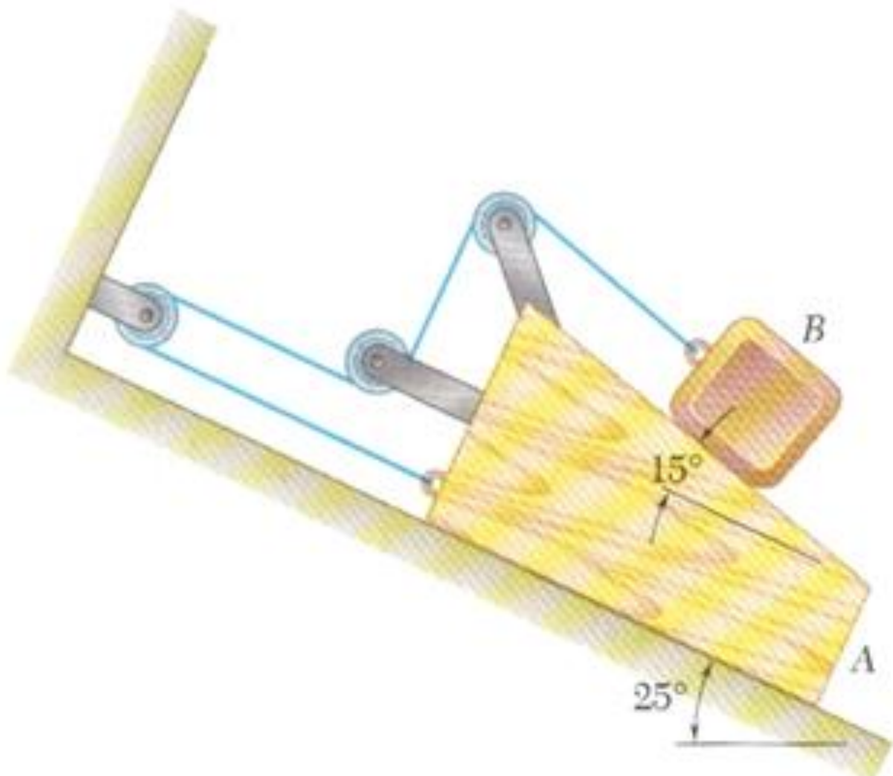


Fig. P11.123

Problem 11.143

11.143 A golfer hits a golf ball from point A with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (*a*) at point A, (*b*) at the highest point of the trajectory.

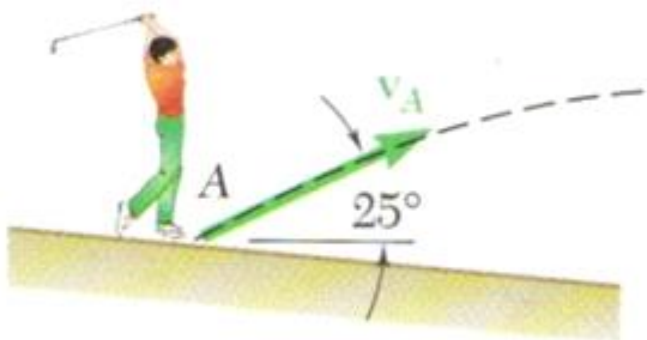


Fig. P11.143

Problem 11.153

11.153 through 11.155 A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet.

11.153 Venus: $g = 8.53\ \text{m/s}^2$, $R = 6161$ km.

Problem 11.171

11.171 For the race car of Prob. 11.167, it was found that it took 0.5 s for the car to travel from the position $\theta = 60°$ to the position $\theta = 35°$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.5-s interval.

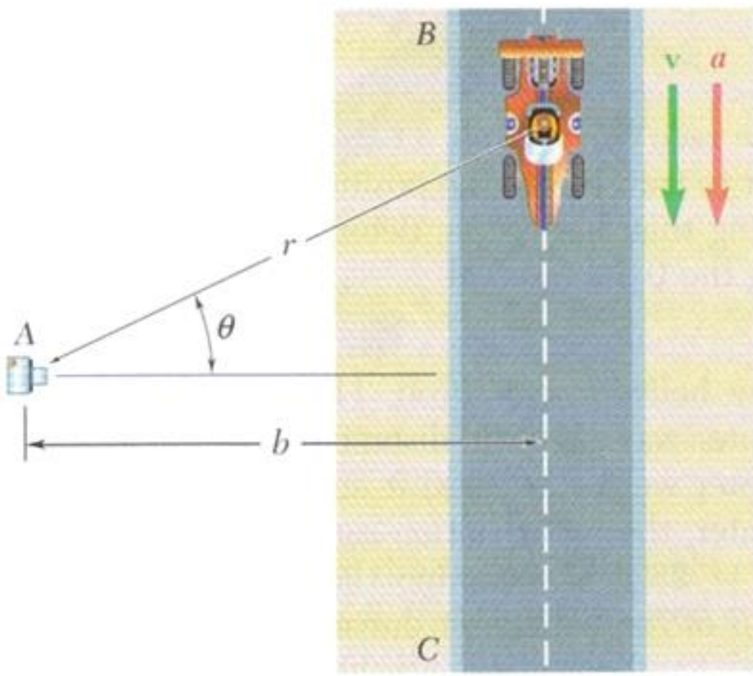


Fig. *P11.167*