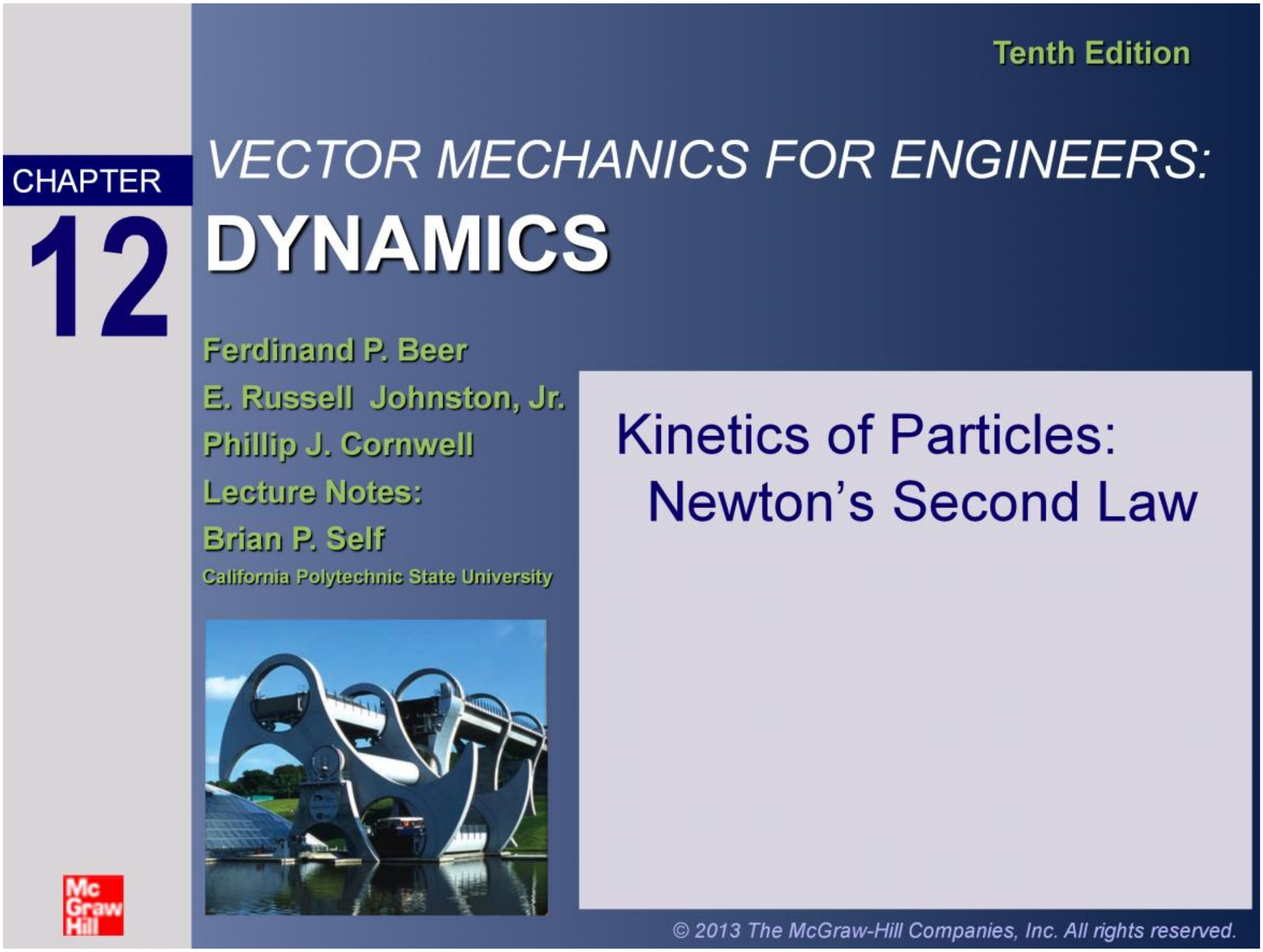

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CHAPTER

12

VECTOR MECHANICS FOR ENGINEERS:
DYNAMICS

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Lecture Notes:
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California Polytechnic State University

Kinetics of Particles:
Newton’s Second Law

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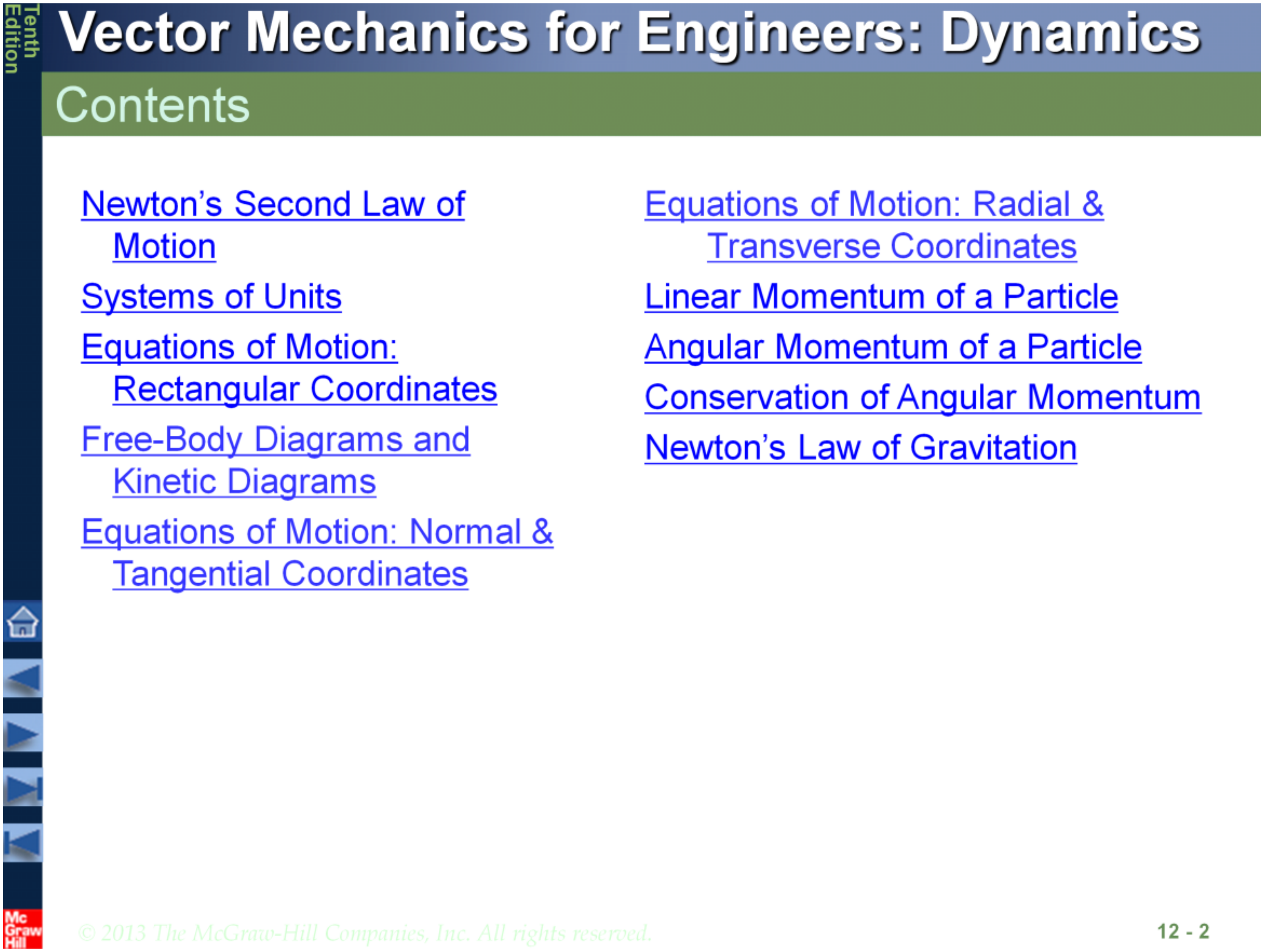

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Vector Mechanics for Engineers: Dynamics

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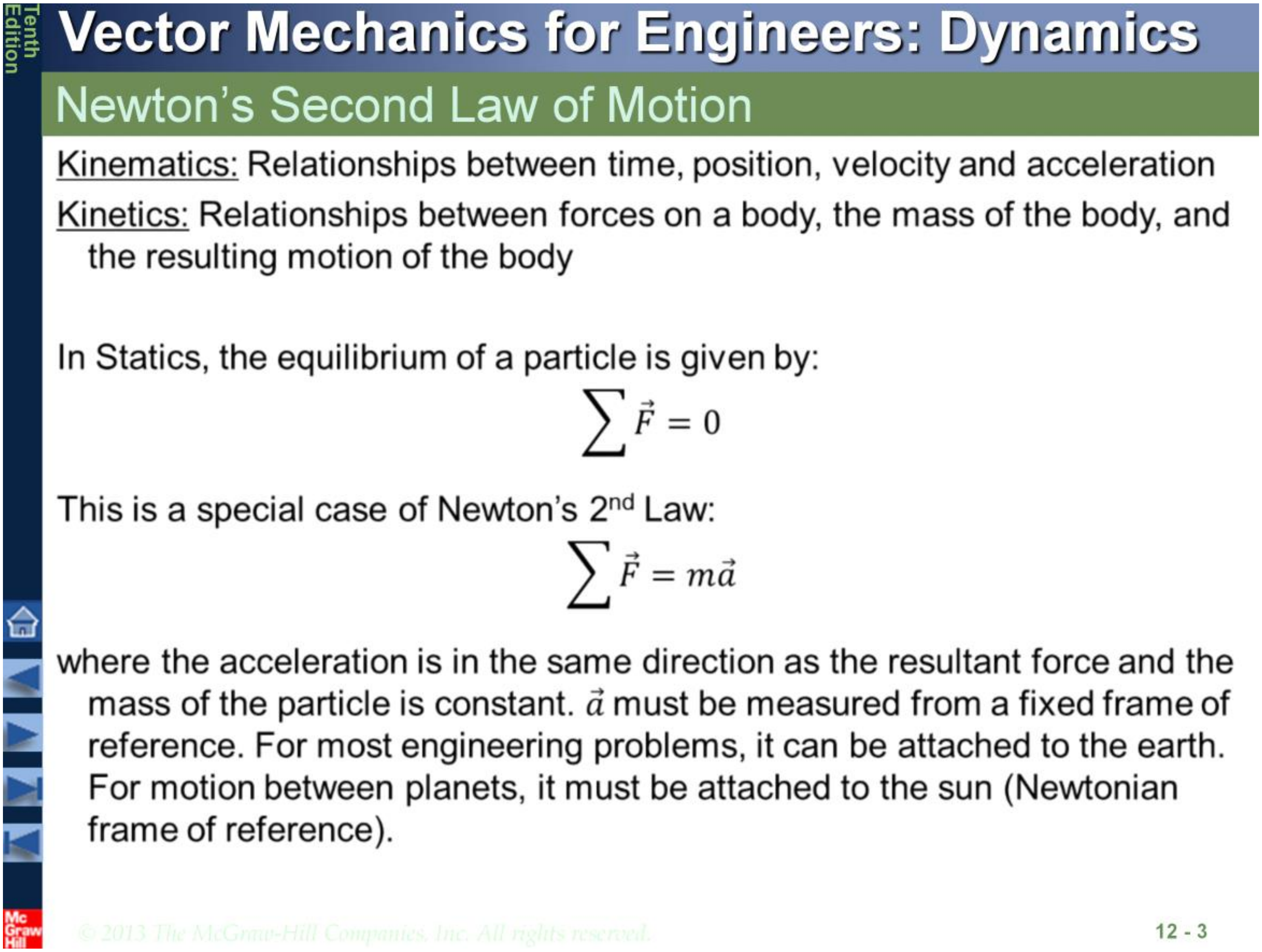
Newton's Second Law of Motion



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Newton’s Second Law of Motion

Kinematics: Relationships between time, position, velocity and acceleration

Kinetics: Relationships between forces on a body, the mass of the body, and the resulting motion of the body

In Statics, the equilibrium of a particle is given by:

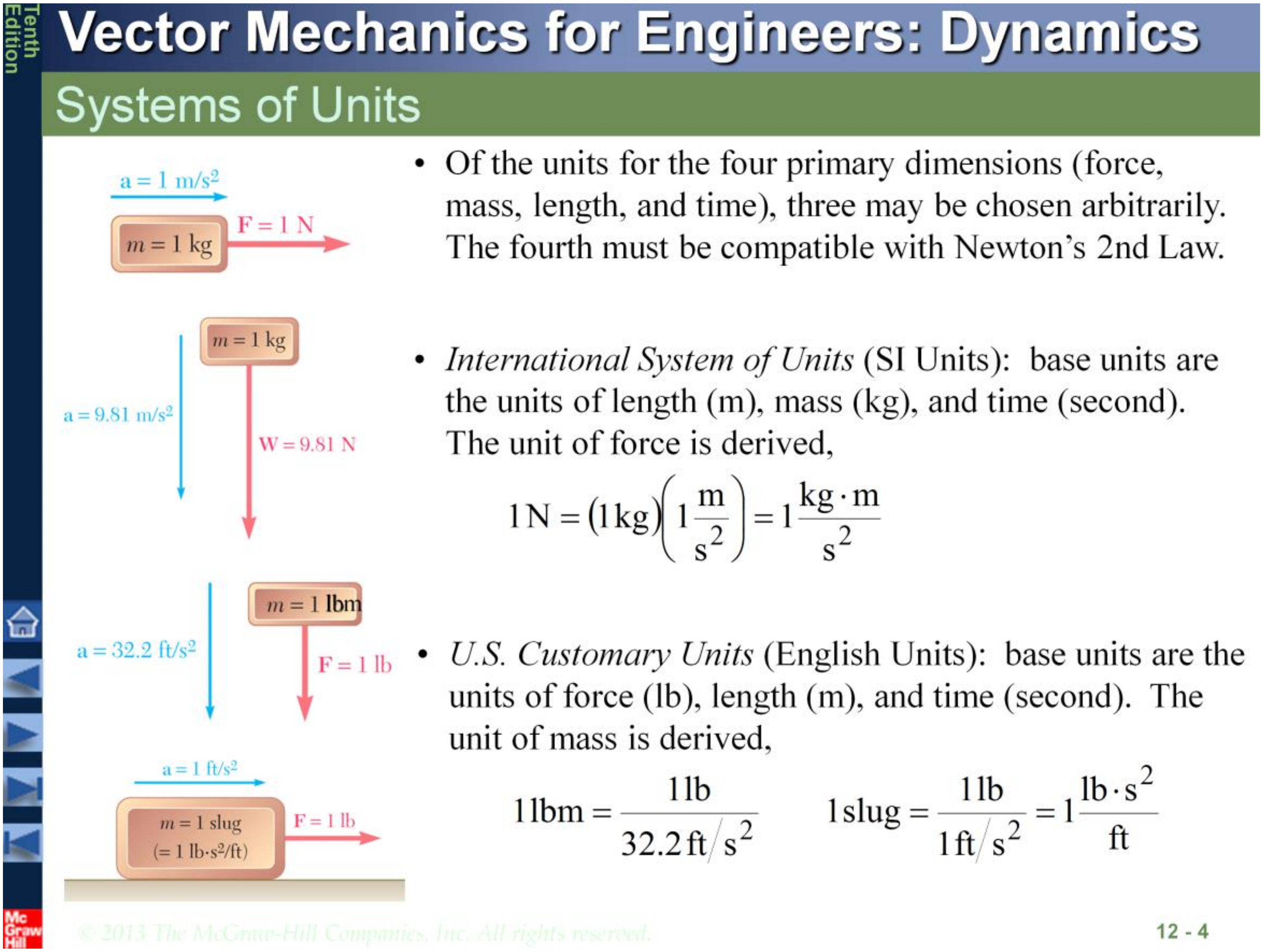
$$\sum \vec{F} = 0$$

This is a special case of Newton’s 2nd Law:

$$\sum \vec{F} = m\vec{a}$$

where the acceleration is in the same direction as the resultant force and the mass of the particle is constant. $\vec{a}$ must be measured from a fixed frame of reference. For most engineering problems, it can be attached to the earth. For motion between planets, it must be attached to the sun (Newtonian frame of reference).

Systems of Units



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Systems of Units

- Of the units for the four primary dimensions (force, mass, length, and time), three may be chosen arbitrarily. The fourth must be compatible with Newton's 2nd Law.

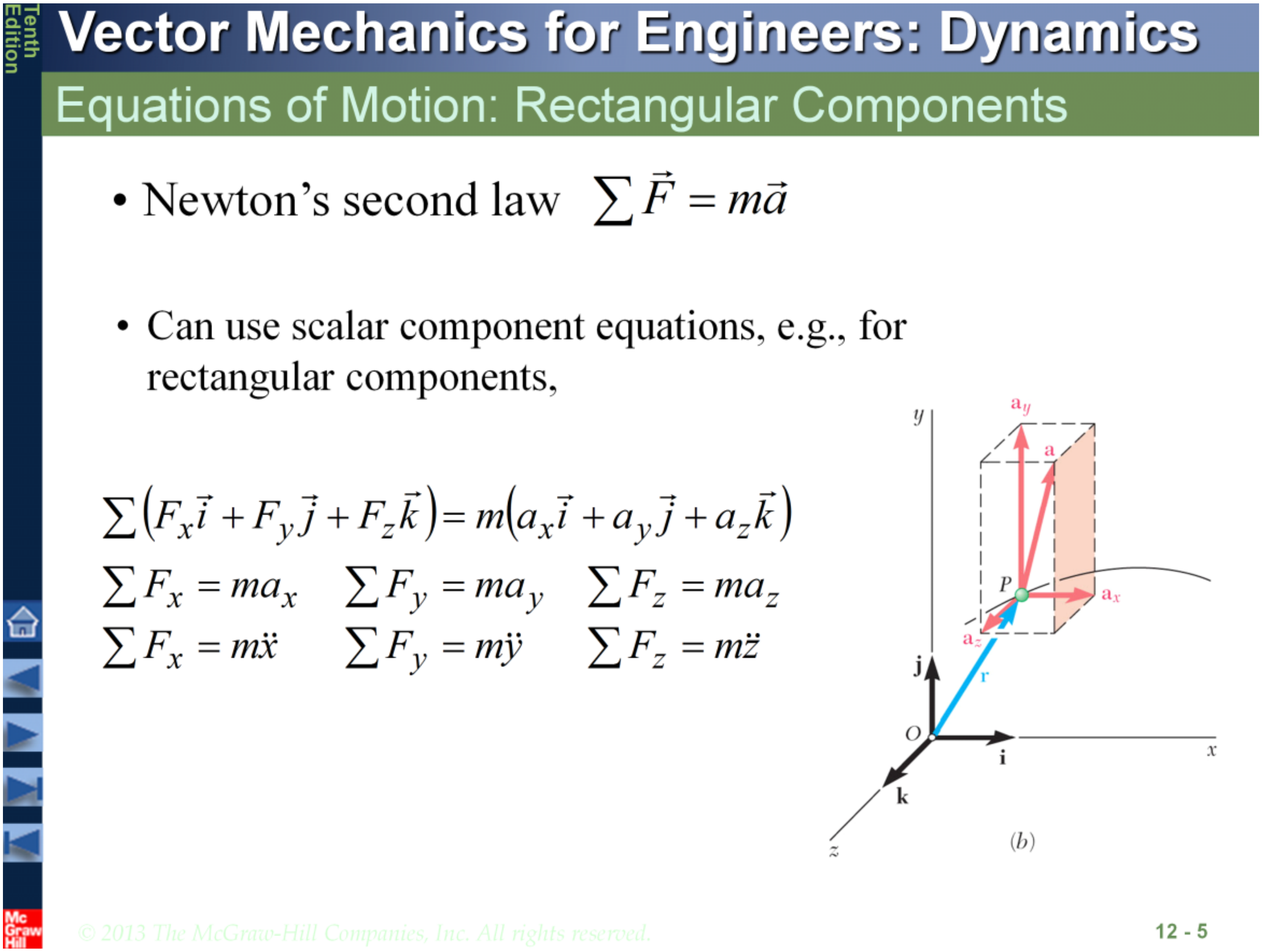
- *International System of Units* (SI Units): base units are the units of length (m), mass (kg), and time (second). The unit of force is derived,

$$1\,\mathrm{N} = (1\,\mathrm{kg})\left(1\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 1\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$$

- *U.S. Customary Units* (English Units): base units are the units of force (lb), length (m), and time (second). The unit of mass is derived,

$$1\,\mathrm{lbm} = \frac{1\,\mathrm{lb}}{32.2\,\mathrm{ft}/\mathrm{s}^2} \qquad 1\,\mathrm{slug} = \frac{1\,\mathrm{lb}}{1\,\mathrm{ft}/\mathrm{s}^2} = 1\frac{\mathrm{lb}\cdot\mathrm{s}^2}{\mathrm{ft}}$$

Equations of Motion: Rectangular Components



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Equations of Motion: Rectangular Components

- Newton's second law $\sum \vec{F} = m\vec{a}$

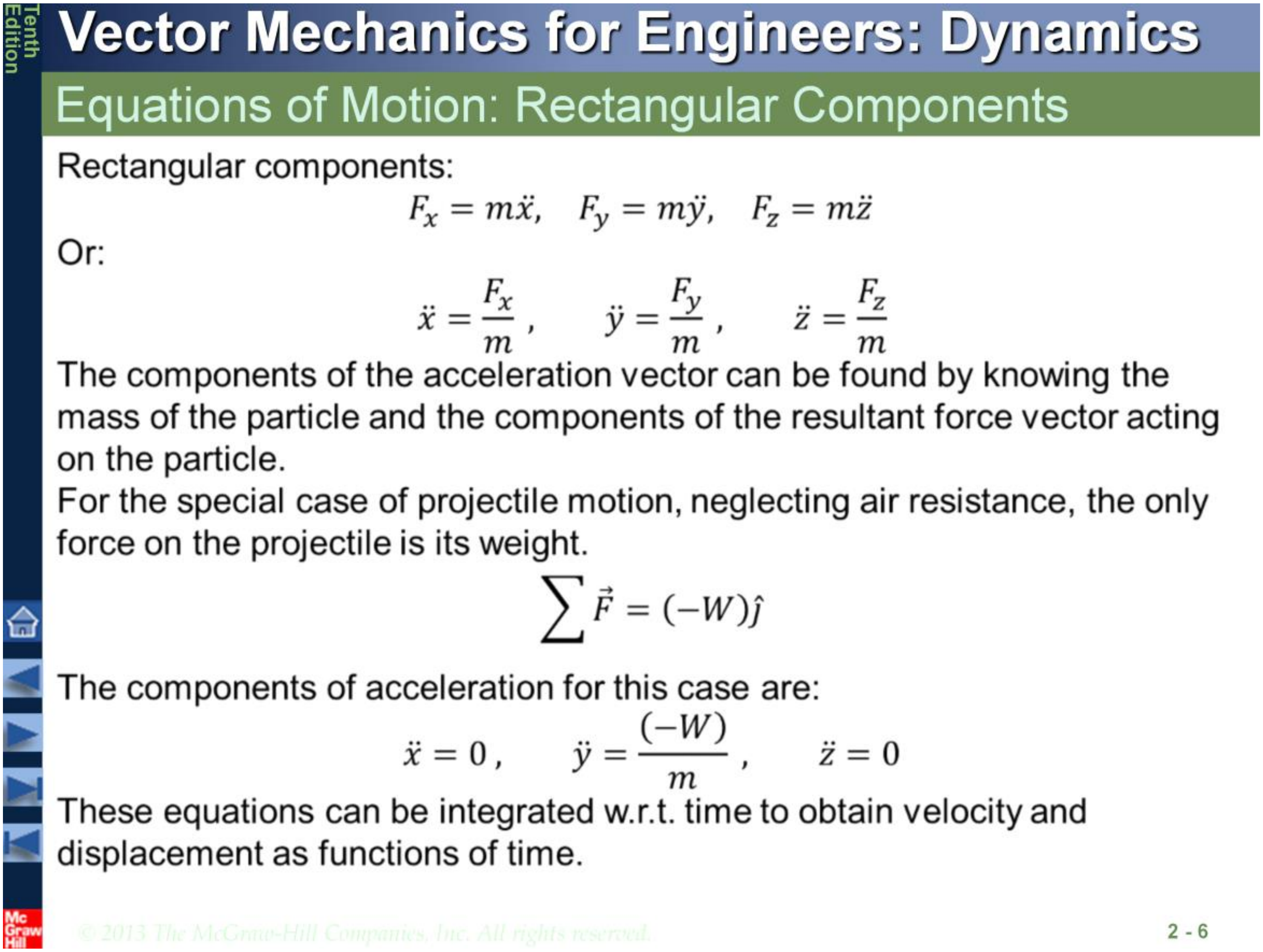
- Can use scalar component equations, e.g., for rectangular components,

$$\sum\left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right) = m\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$\sum F_x = m\ddot{x} \qquad \sum F_y = m\ddot{y} \qquad \sum F_z = m\ddot{z}$$

Equations of Motion: Rectangular Components



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Vector Mechanics for Engineers: Dynamics

Equations of Motion: Rectangular Components

Rectangular components:

$$F_x = m\ddot{x}, \quad F_y = m\ddot{y}, \quad F_z = m\ddot{z}$$

Or:

$$\ddot{x} = \frac{F_x}{m}, \qquad \ddot{y} = \frac{F_y}{m}, \qquad \ddot{z} = \frac{F_z}{m}$$

The components of the acceleration vector can be found by knowing the mass of the particle and the components of the resultant force vector acting on the particle.

For the special case of projectile motion, neglecting air resistance, the only force on the projectile is its weight.

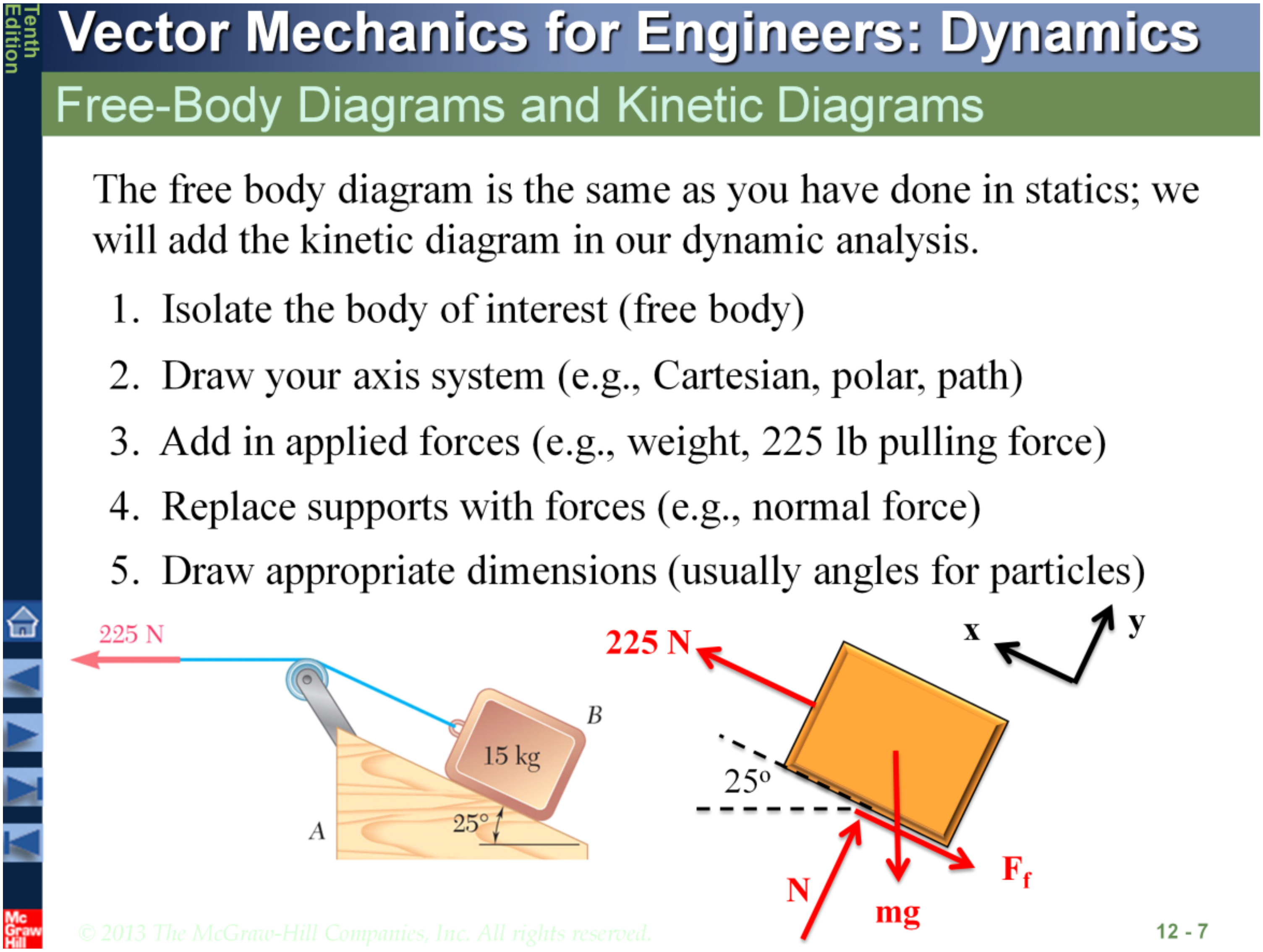
$$\sum \vec{F} = (-W)\hat{\jmath}$$

The components of acceleration for this case are:

$$\ddot{x} = 0, \qquad \ddot{y} = \frac{(-W)}{m}, \qquad \ddot{z} = 0$$

These equations can be integrated w.r.t. time to obtain velocity and displacement as functions of time.

Free-Body Diagrams and Kinetic Diagrams



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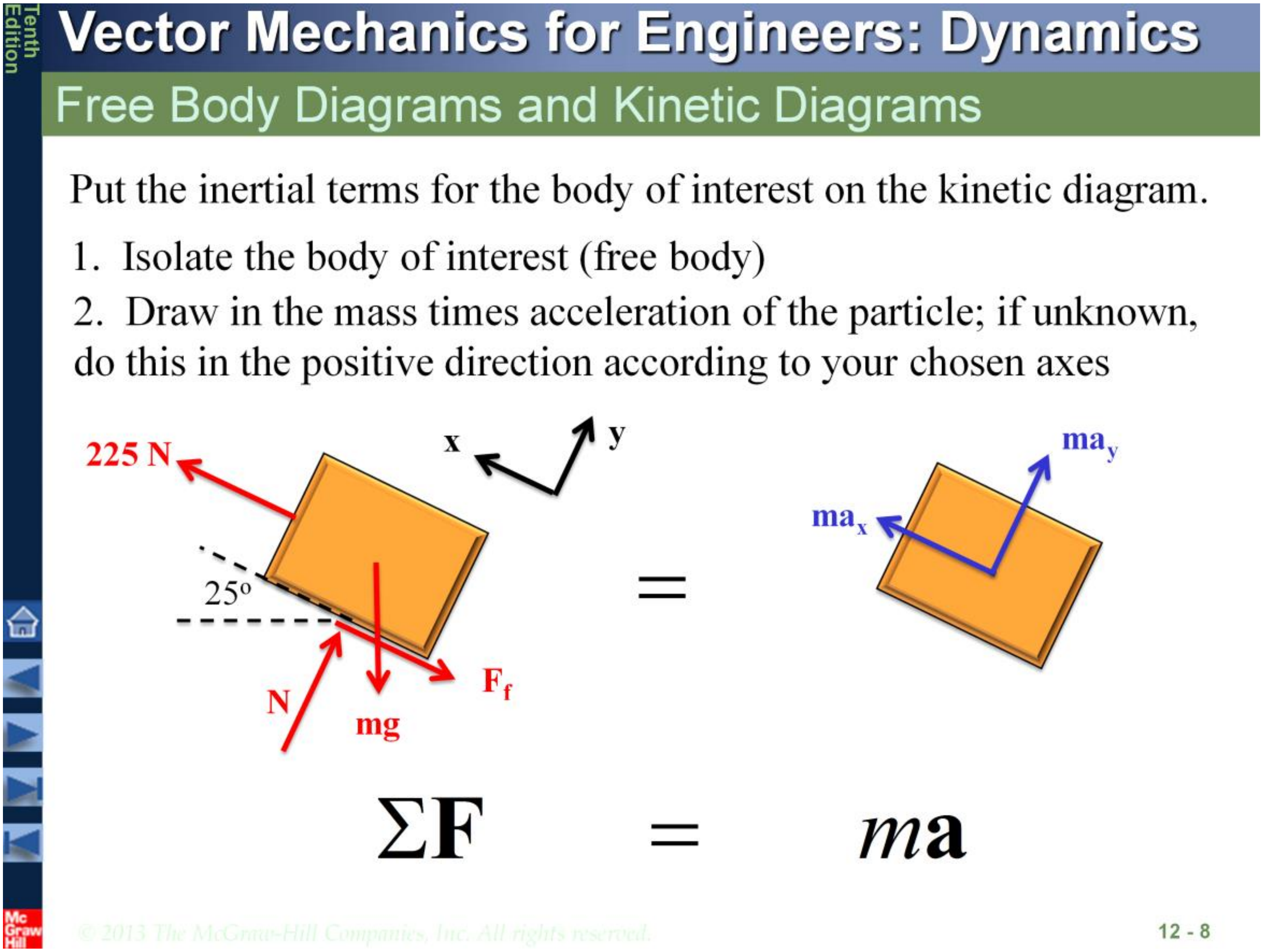
Free-Body Diagrams and Kinetic Diagrams

The free body diagram is the same as you have done in statics; we will add the kinetic diagram in our dynamic analysis.

1. Isolate the body of interest (free body)
2. Draw your axis system (e.g., Cartesian, polar, path)
3. Add in applied forces (e.g., weight, 225 lb pulling force)
4. Replace supports with forces (e.g., normal force)
5. Draw appropriate dimensions (usually angles for particles)

Free-Body Diagrams and Kinetic Diagrams



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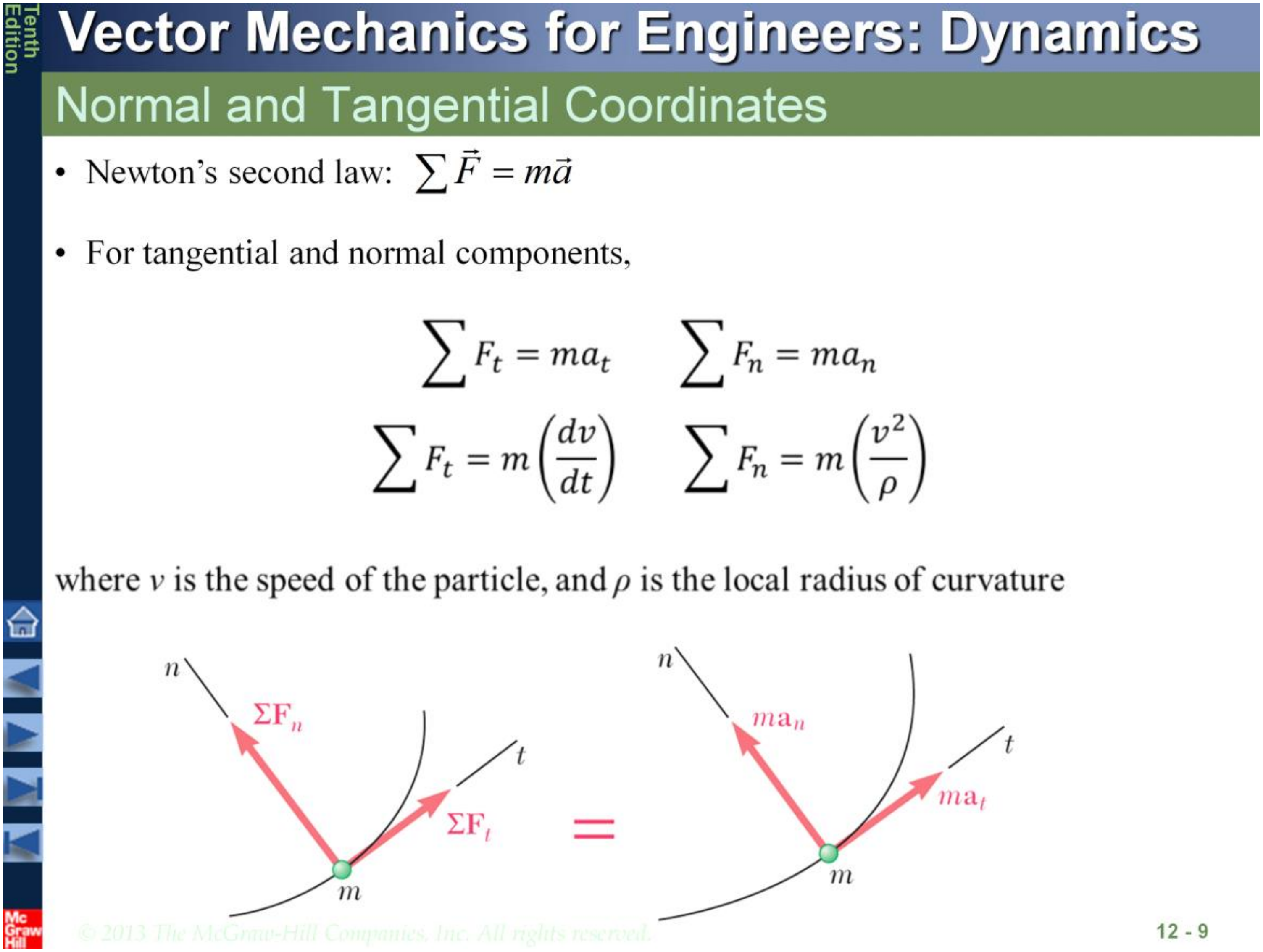
Free Body Diagrams and Kinetic Diagrams

Put the inertial terms for the body of interest on the kinetic diagram.

1. Isolate the body of interest (free body)
2. Draw in the mass times acceleration of the particle; if unknown, do this in the positive direction according to your chosen axes

$$\Sigma\mathbf{F} = m\mathbf{a}$$

Normal and Tangential Coordinates



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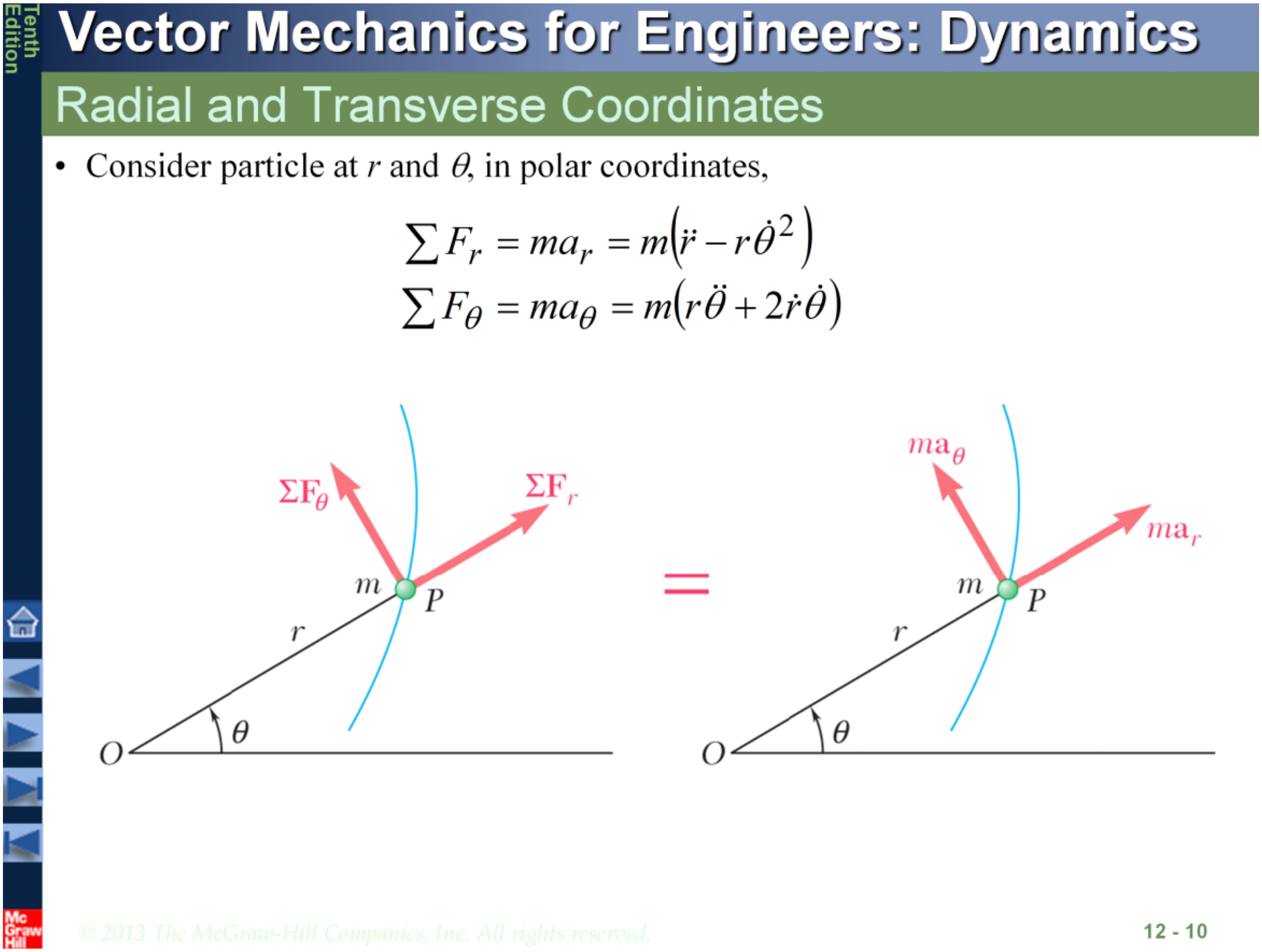
Normal and Tangential Coordinates

- Newton's second law: $\sum \vec{F} = m\vec{a}$
- For tangential and normal components,

$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$

$$\sum F_t = m\left(\frac{dv}{dt}\right) \qquad \sum F_n = m\left(\frac{v^2}{\rho}\right)$$

where v is the speed of the particle, and ρ is the local radius of curvature



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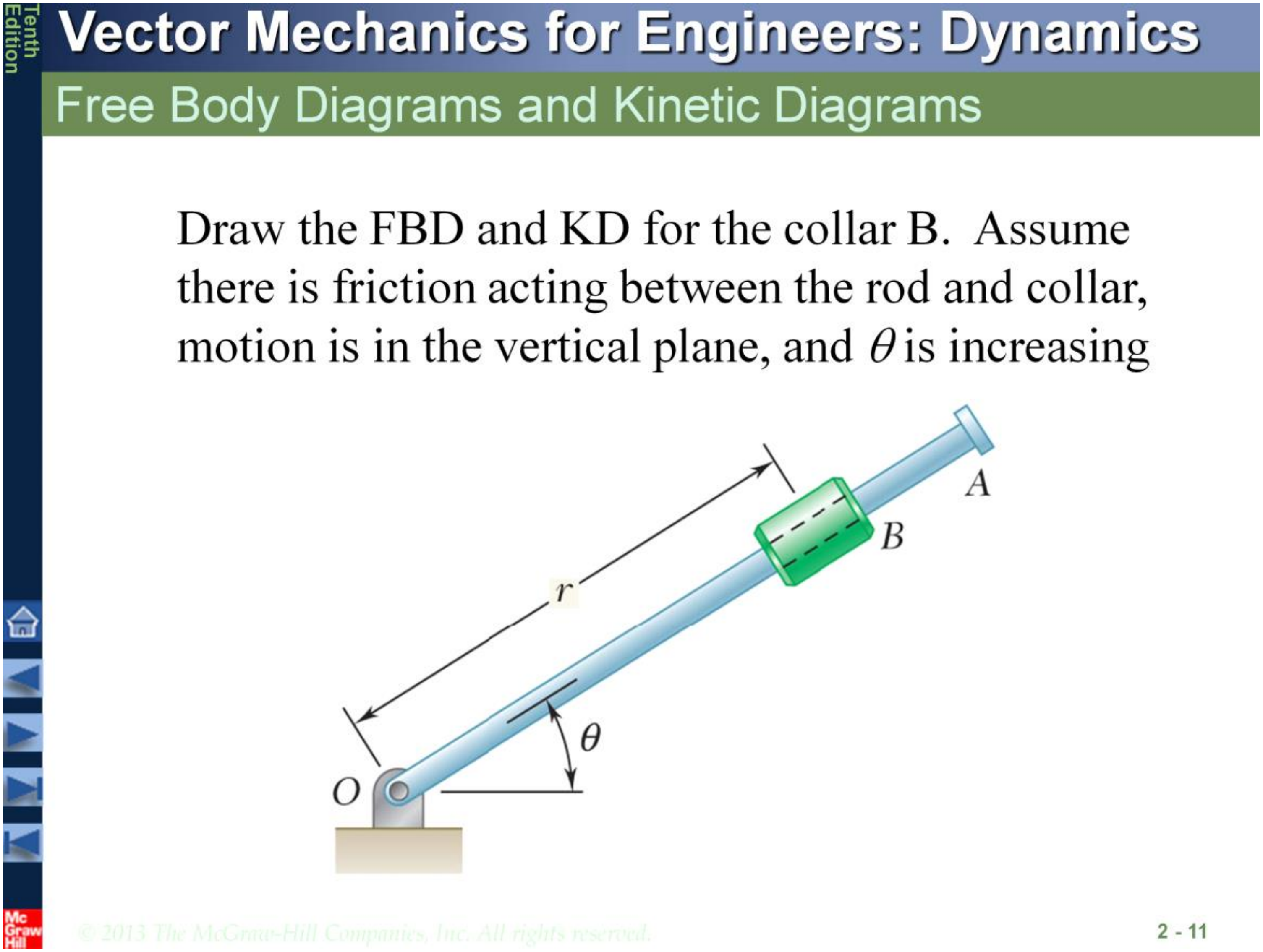
Radial and Transverse Coordinates

- Consider particle at r and θ, in polar coordinates,

$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$
$$\sum F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

Free-Body Diagrams and Kinetic Diagrams



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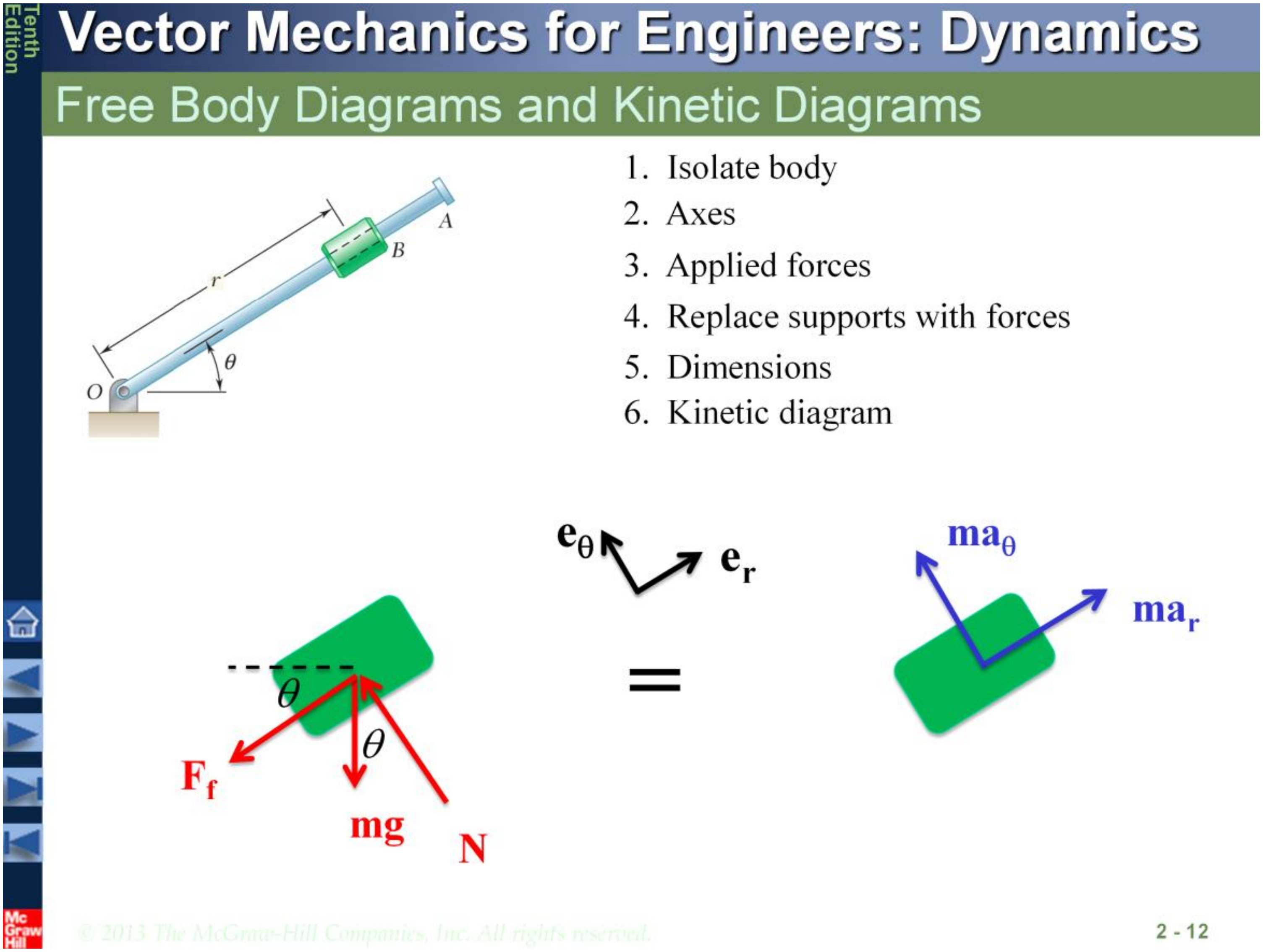
Free Body Diagrams and Kinetic Diagrams

Draw the FBD and KD for the collar B. Assume there is friction acting between the rod and collar, motion is in the vertical plane, and θ is increasing

Free-Body Diagrams and Kinetic Diagrams



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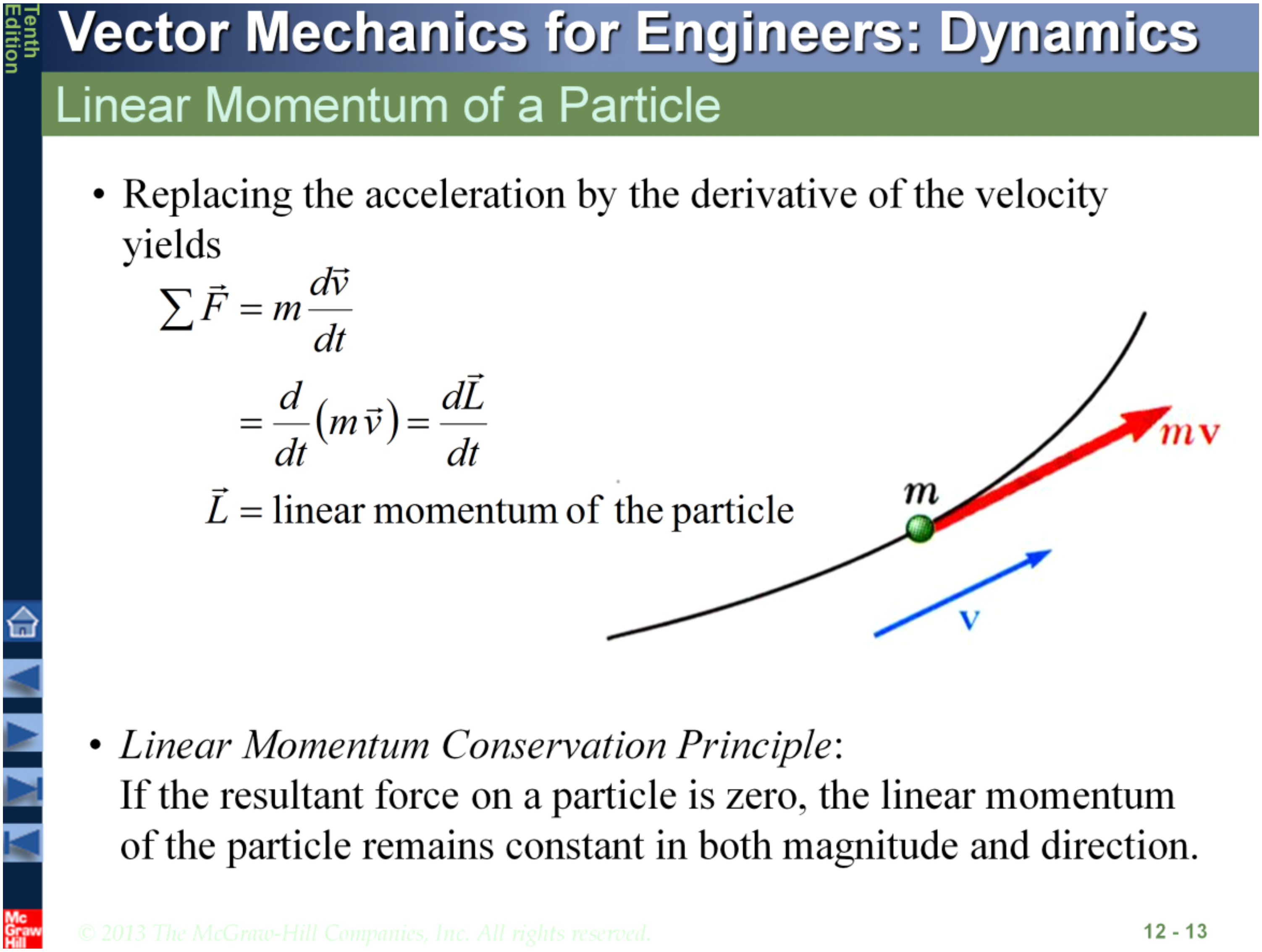
Free Body Diagrams and Kinetic Diagrams

1. Isolate body
2. Axes
3. Applied forces
4. Replace supports with forces
5. Dimensions
6. Kinetic diagram

Linear Momentum of a Particle



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Linear Momentum of a Particle

- Replacing the acceleration by the derivative of the velocity yields

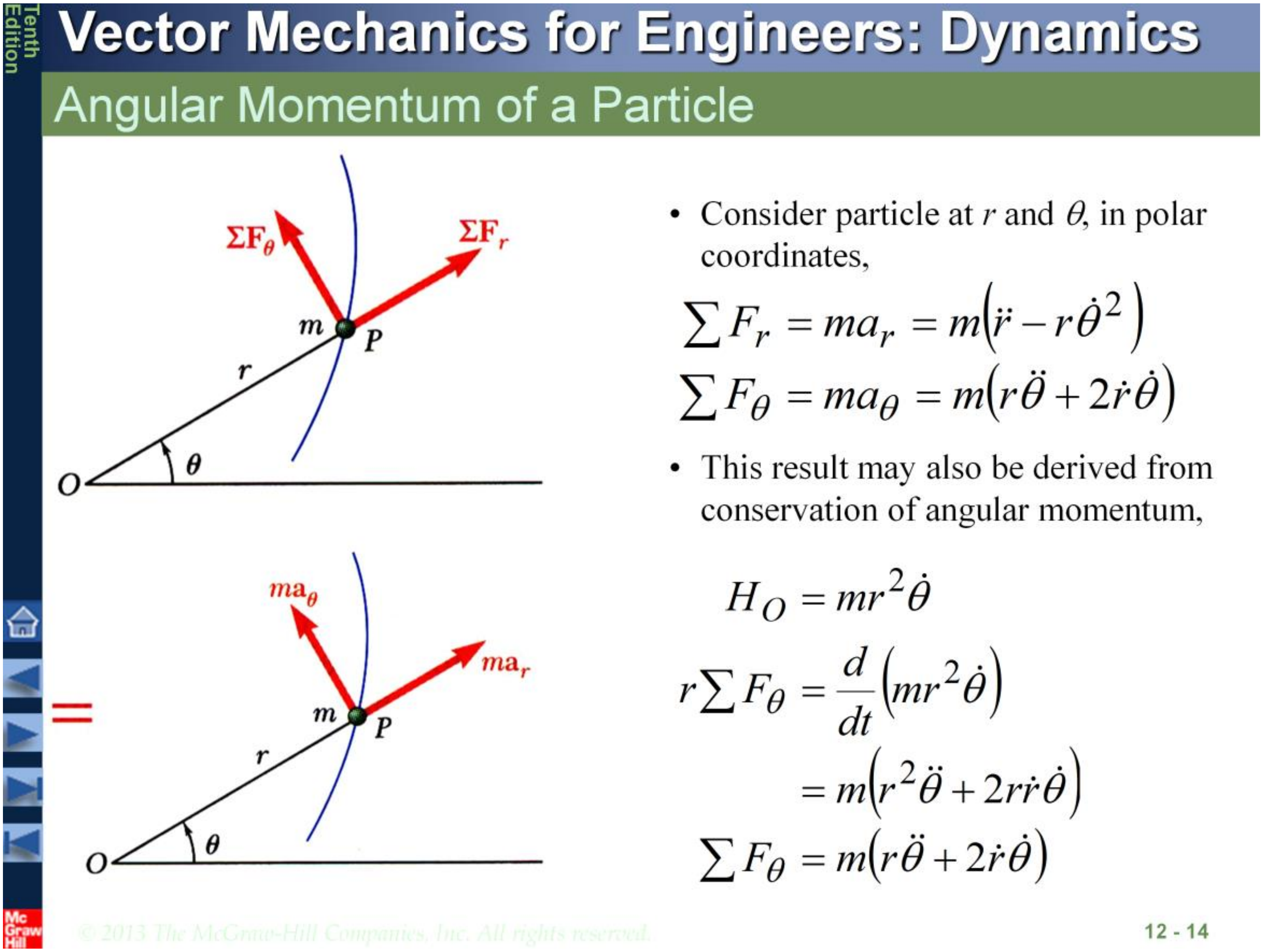
$$\sum \vec{F} = m\frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt}(m\vec{v}) = \frac{d\vec{L}}{dt}$$

$\vec{L}$ = linear momentum of the particle

- *Linear Momentum Conservation Principle*:
  If the resultant force on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction.

Angular Momentum of a Particle



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Angular Momentum of a Particle

- Consider particle at *r* and *θ*, in polar coordinates,

$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\sum F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

- This result may also be derived from conservation of angular momentum,

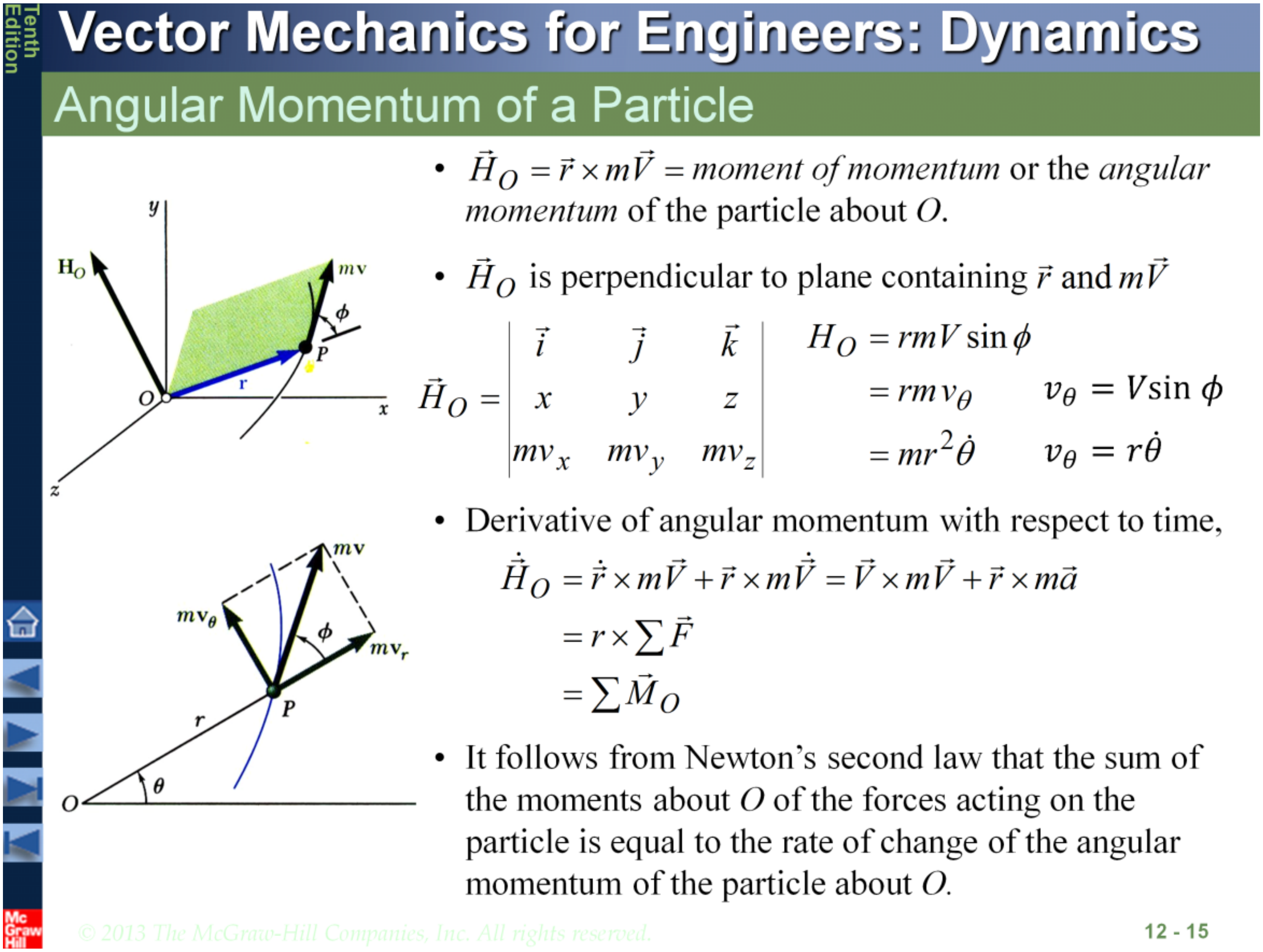
$$H_O = mr^2\dot{\theta}$$

$$r\sum F_\theta = \frac{d}{dt}\left(mr^2\dot{\theta}\right)$$

$$= m\left(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta}\right)$$

$$\sum F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

Angular Momentum of a Particle



Vector Mechanics for Engineers: Dynamics

Angular Momentum of a Particle

- $\vec{H}_O = \vec{r} \times m\vec{V}$ = *moment of momentum* or the *angular momentum* of the particle about O.
- $\vec{H}_O$ is perpendicular to plane containing $\vec{r}$ and $m\vec{V}$

$$\vec{H}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

$$H_O = rmV\sin\phi$$

$$= rmv_\theta \qquad v_\theta = V\sin\phi$$

$$= mr^2\dot{\theta} \qquad v_\theta = r\dot{\theta}$$

- Derivative of angular momentum with respect to time,

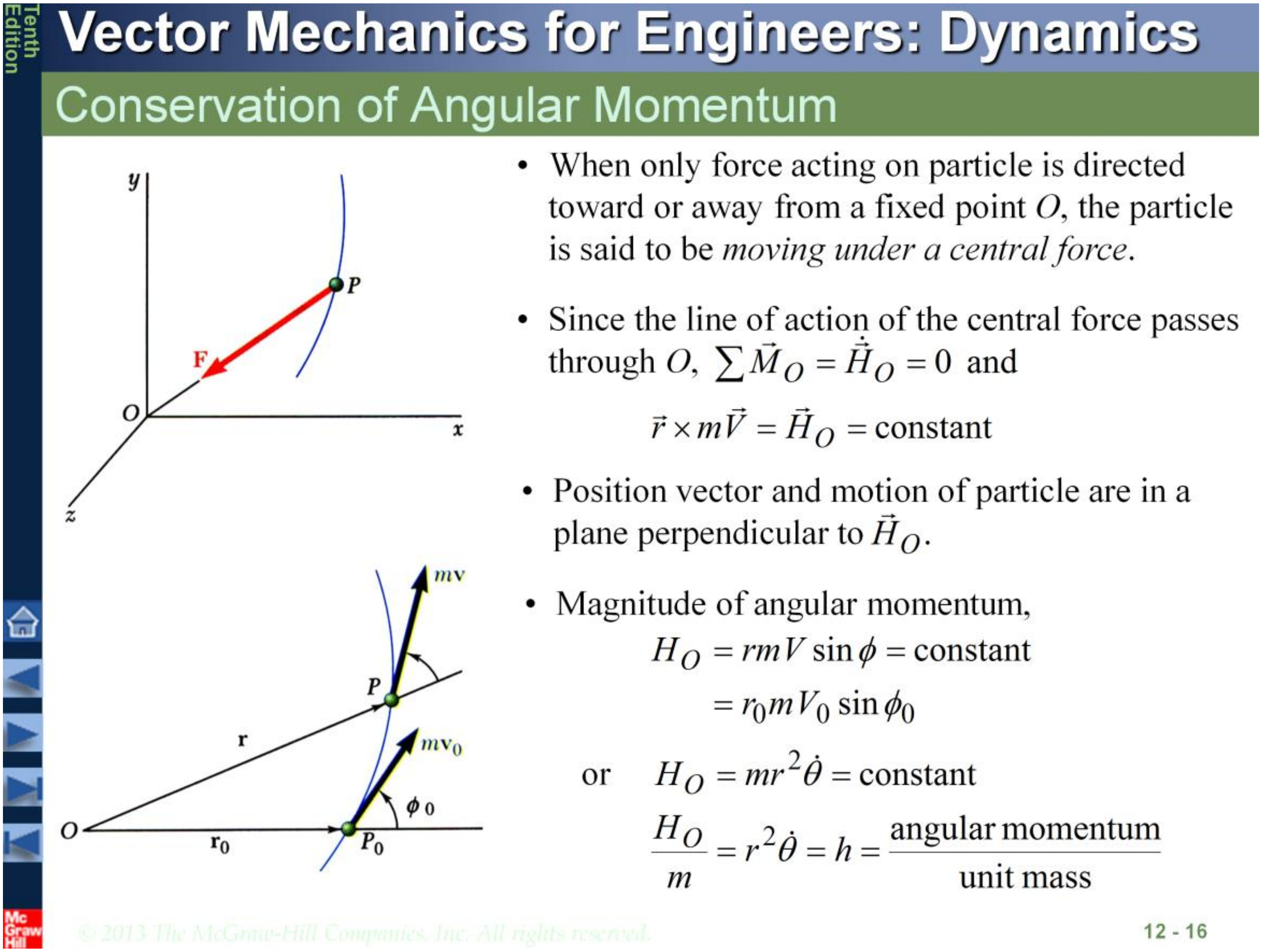
$$\dot{\vec{H}}_O = \dot{\vec{r}} \times m\vec{V} + \vec{r} \times m\dot{\vec{V}} = \vec{V} \times m\vec{V} + \vec{r} \times m\vec{a}$$

$$= r \times \sum \vec{F}$$

$$= \sum \vec{M}_O$$

- It follows from Newton's second law that the sum of the moments about O of the forces acting on the particle is equal to the rate of change of the angular momentum of the particle about O.

Conservation of Angular Momentum



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Conservation of Angular Momentum

- When only force acting on particle is directed toward or away from a fixed point O, the particle is said to be *moving under a central force*.
- Since the line of action of the central force passes through O, $\sum \vec{M}_O = \dot{\vec{H}}_O = 0$ and

$$\vec{r} \times m\vec{V} = \vec{H}_O = \text{constant}$$

- Position vector and motion of particle are in a plane perpendicular to $\vec{H}_O$.
- Magnitude of angular momentum,

$$H_O = rmV\sin\phi = \text{constant}$$

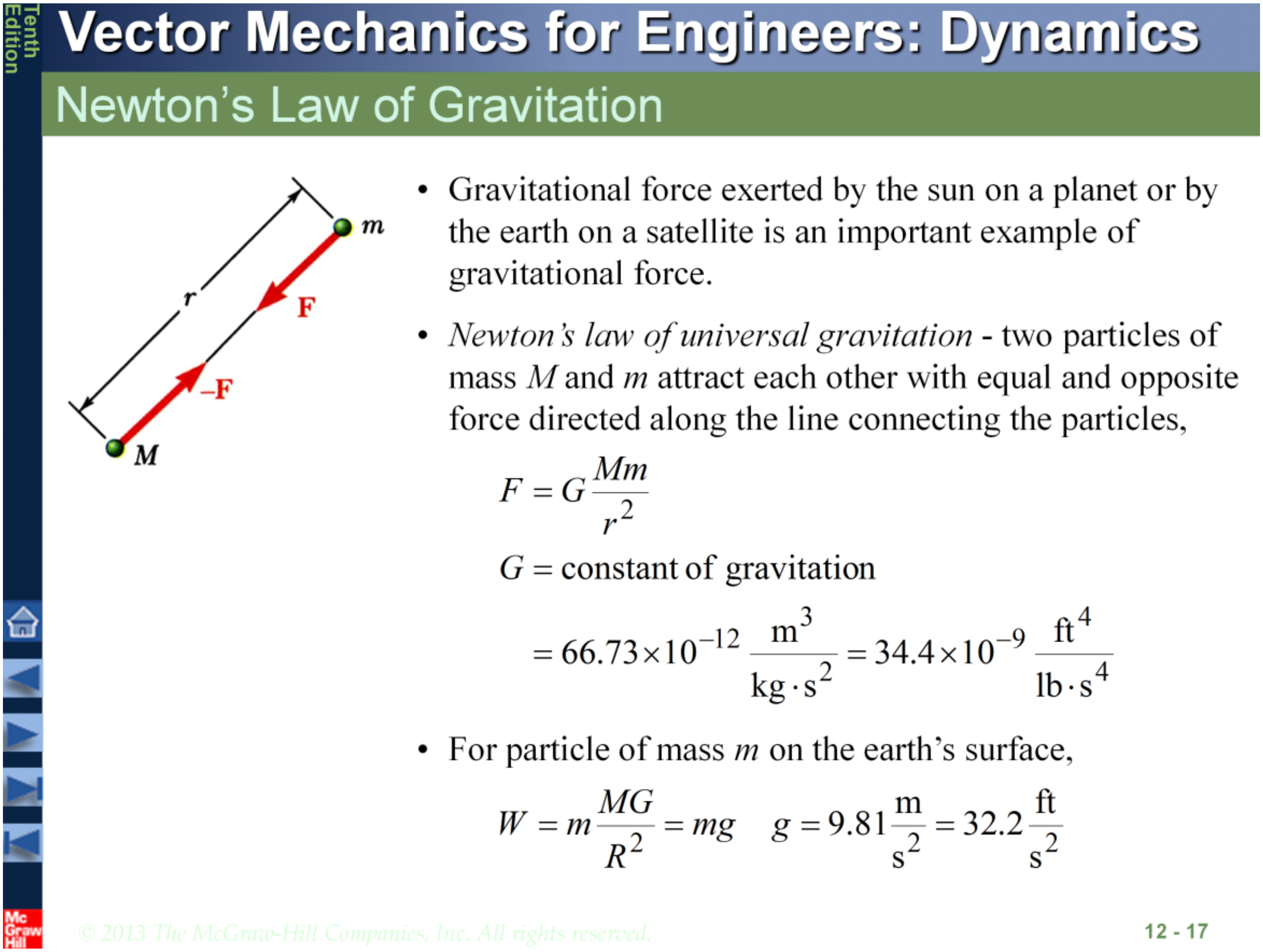
$$= r_0 m V_0 \sin\phi_0$$

or

$$H_O = mr^2\dot{\theta} = \text{constant}$$

$$\frac{H_O}{m} = r^2\dot{\theta} = h = \frac{\text{angular momentum}}{\text{unit mass}}$$

Newton's Law of Gravitation



Vector Mechanics for Engineers: Dynamics

Newton's Law of Gravitation

- Gravitational force exerted by the sun on a planet or by the earth on a satellite is an important example of gravitational force.
- *Newton's law of universal gravitation* - two particles of mass M and m attract each other with equal and opposite force directed along the line connecting the particles,

$$F = G\frac{Mm}{r^2}$$

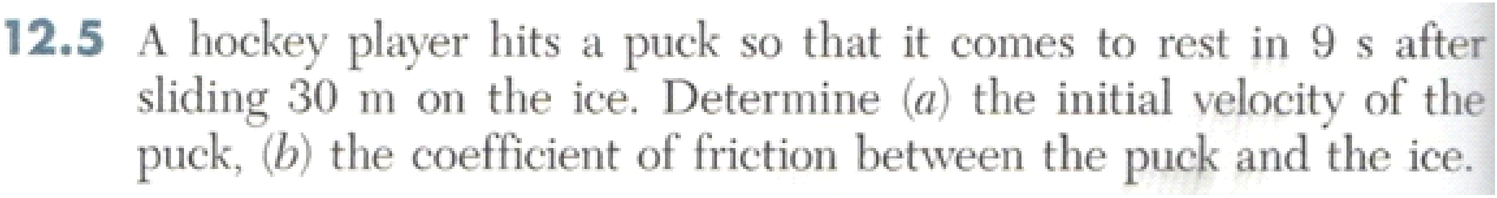
G = constant of gravitation

$$= 66.73\times10^{-12}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2} = 34.4\times10^{-9}\,\frac{\text{ft}^4}{\text{lb}\cdot\text{s}^4}$$

- For particle of mass m on the earth's surface,

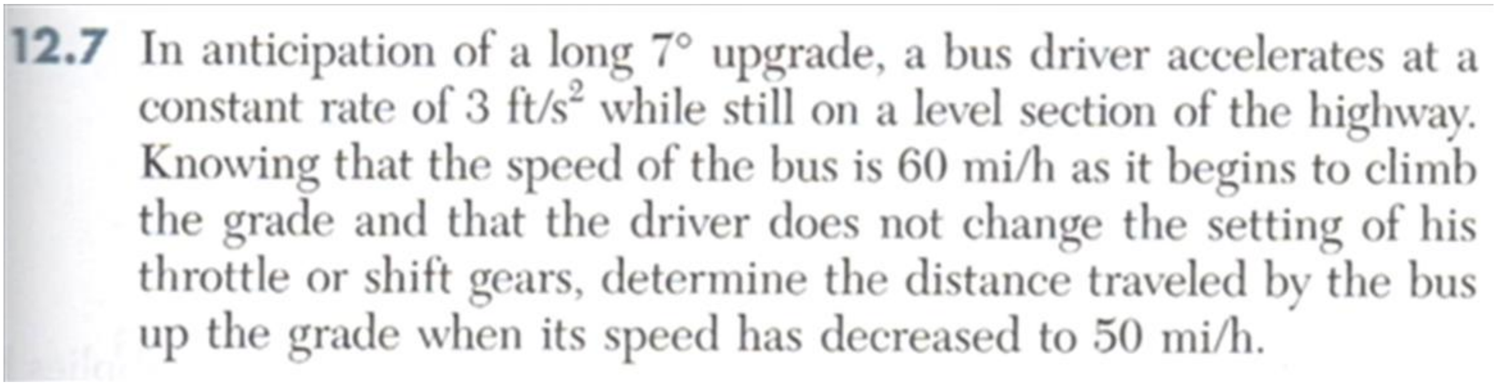
$$W = m\frac{MG}{R^2} = mg \qquad g = 9.81\,\frac{\text{m}}{\text{s}^2} = 32.2\,\frac{\text{ft}}{\text{s}^2}$$

Problem 12-5



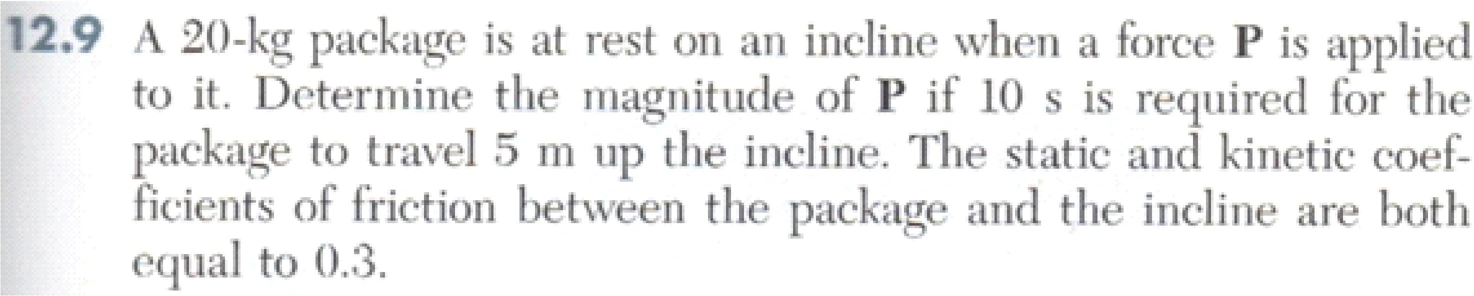
12.5 A hockey player hits a puck so that it comes to rest in 9 s after sliding 30 m on the ice. Determine (*a*) the initial velocity of the puck, (*b*) the coefficient of friction between the puck and the ice.

Problem 12-7



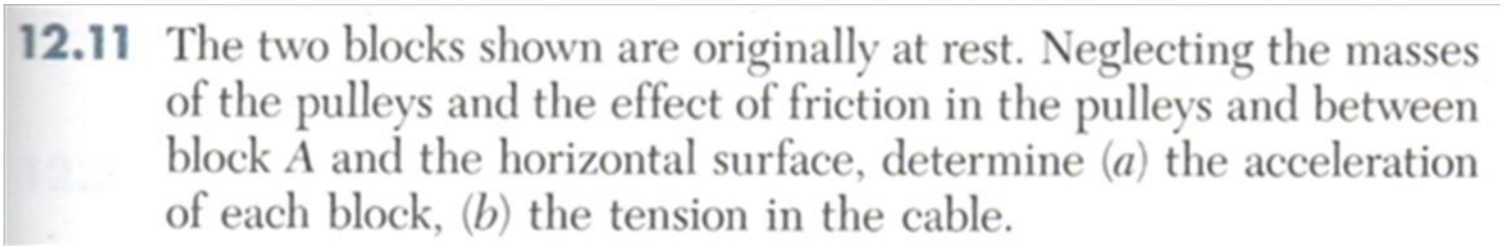
12.7 In anticipation of a long 7° upgrade, a bus driver accelerates at a constant rate of 3 ft/s^2 while still on a level section of the highway. Knowing that the speed of the bus is 60 mi/h as it begins to climb the grade and that the driver does not change the setting of his throttle or shift gears, determine the distance traveled by the bus up the grade when its speed has decreased to 50 mi/h.

Problem 12-9

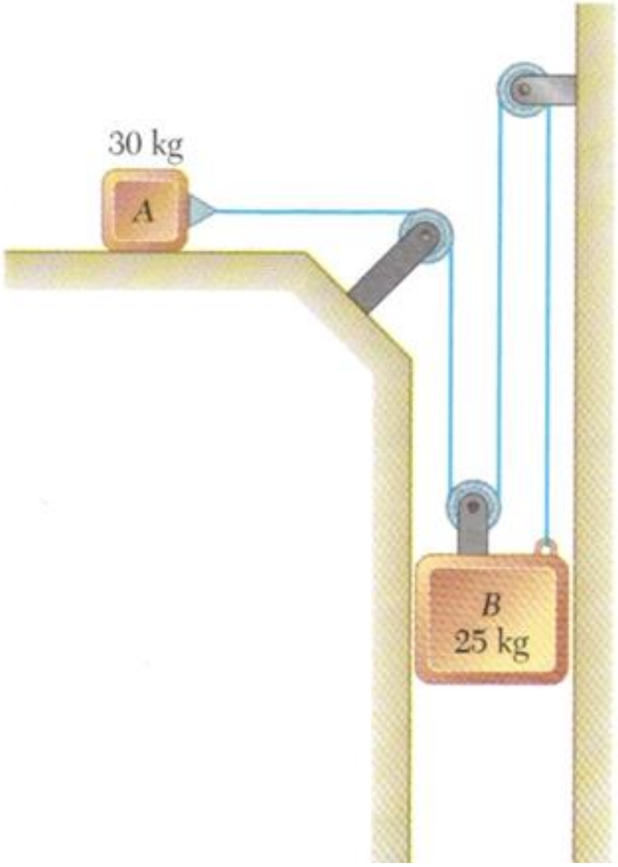


12.9 A 20-kg package is at rest on an incline when a force **P** is applied to it. Determine the magnitude of **P** if 10 s is required for the package to travel 5 m up the incline. The static and kinetic coefficients of friction between the package and the incline are both equal to 0.3.

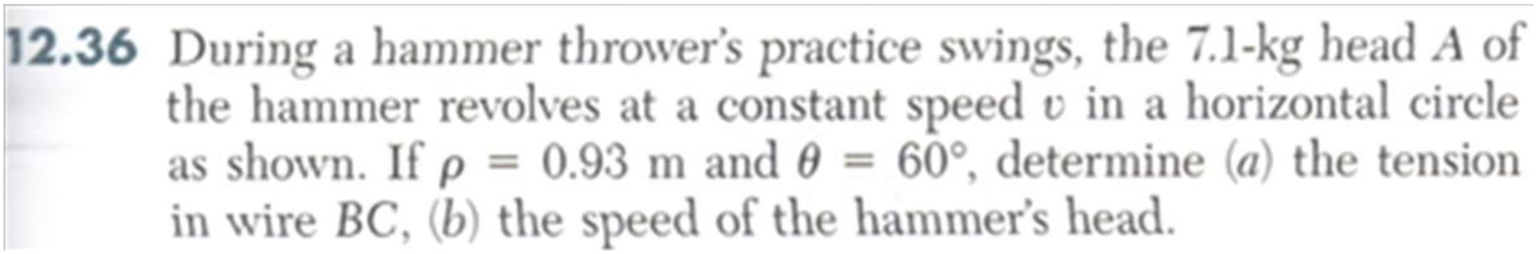
Problem 12-11



12.11 The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block *A* and the horizontal surface, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.



Problem 12-36



12.36 During a hammer thrower's practice swings, the 7.1-kg head A of the hammer revolves at a constant speed v in a horizontal circle as shown. If $\rho = 0.93$ m and $\theta = 60°$, determine (*a*) the tension in wire BC, (*b*) the speed of the hammer's head.

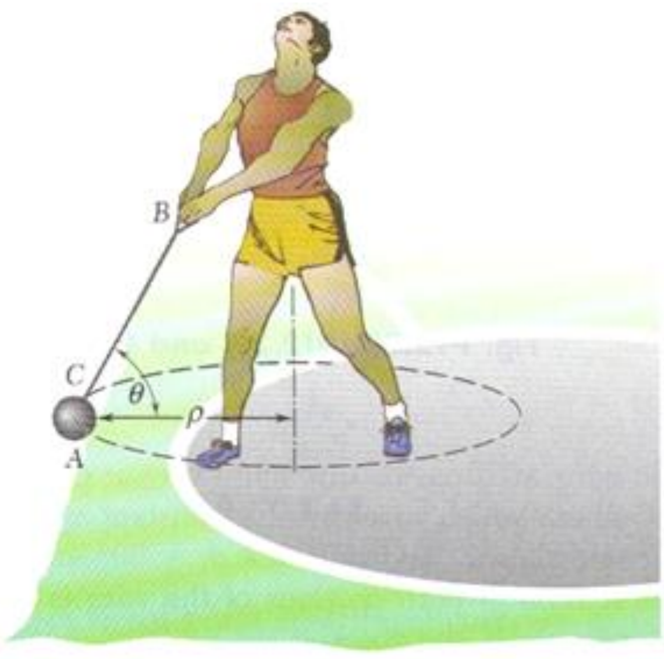
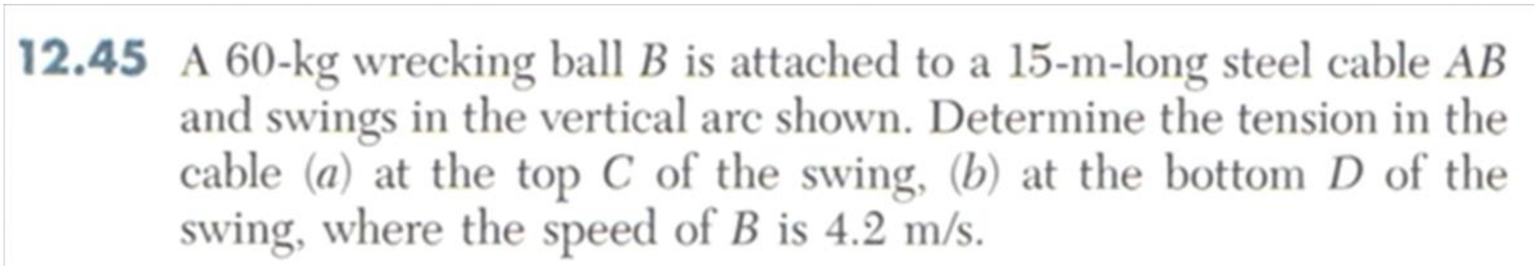


Fig. P12.36

Problem 12-45



12.45 A 60-kg wrecking ball *B* is attached to a 15-m-long steel cable *AB* and swings in the vertical arc shown. Determine the tension in the cable (*a*) at the top *C* of the swing, (*b*) at the bottom *D* of the swing, where the speed of *B* is 4.2 m/s.

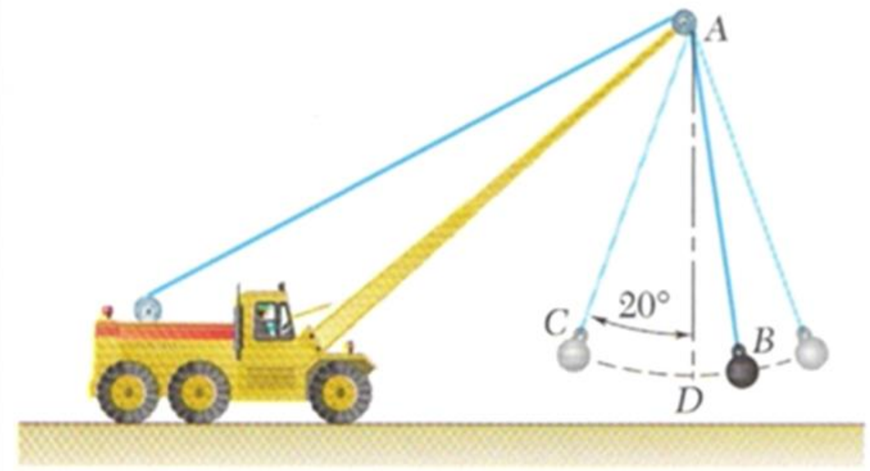
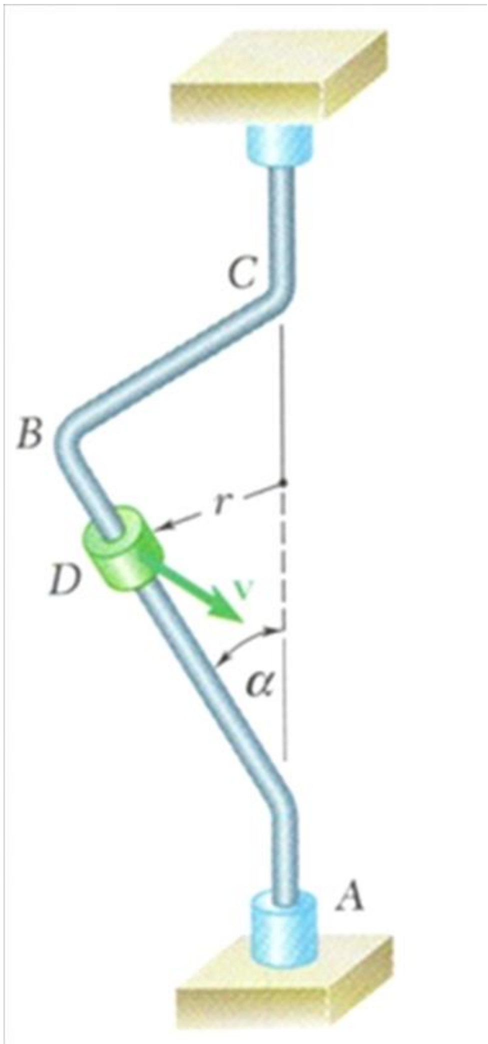


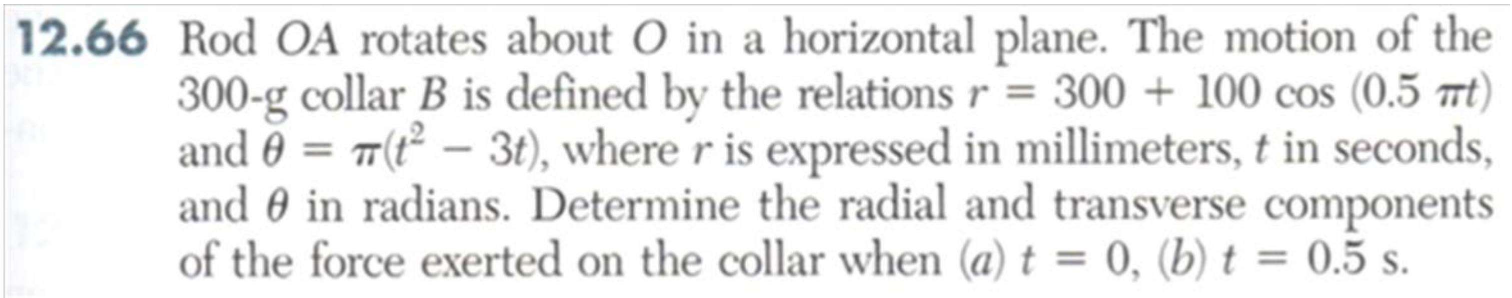
Fig. P12.45

Problem 12-55

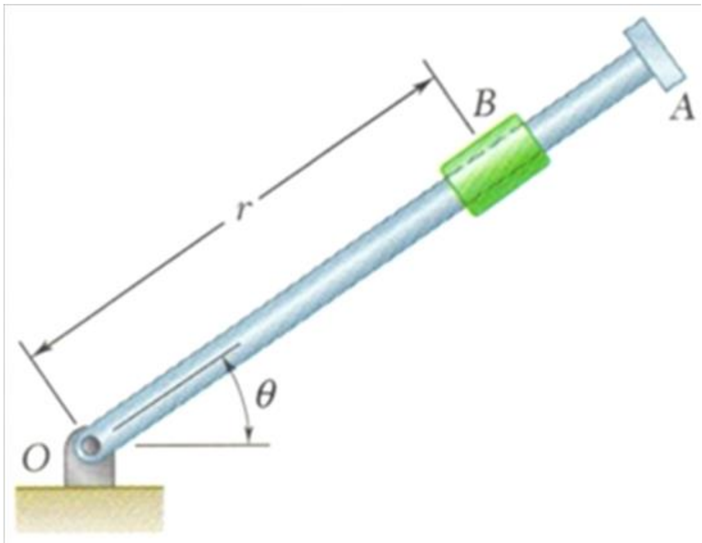
12.55 A small, 300-g collar *D* can slide on portion *AB* of a rod which is bent as shown. Knowing that $\alpha = 40°$ and that the rod rotates about the vertical *AC* at a constant rate of 5 rad/s, determine the value of *r* for which the collar will not slide on the rod if the effect of friction between the rod and the collar is neglected.



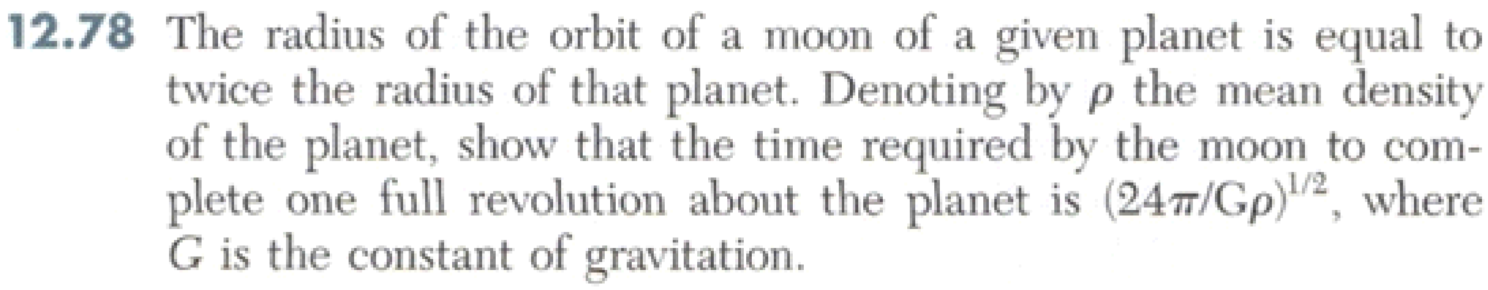
Problem 12-66



12.66 Rod *OA* rotates about *O* in a horizontal plane. The motion of the 300-g collar *B* is defined by the relations $r = 300 + 100\cos(0.5\pi t)$ and $\theta = \pi(t^2 - 3t)$, where r is expressed in millimeters, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the collar when (*a*) $t = 0$, (*b*) $t = 0.5$ s.



Problem 12-78



12.78 The radius of the orbit of a moon of a given planet is equal to twice the radius of that planet. Denoting by ρ the mean density of the planet, show that the time required by the moon to complete one full revolution about the planet is $(24\pi/G\rho)^{1/2}$, where G is the constant of gravitation.