SO FAR, WE'VE LEARNED HOW TO RELATE TIME, POSITION, VELOCITY AND ACCELERATION USING KINEMATICS AND ~~THE~~ NEWTON'S SECOND LAW $\vec{F} = m\vec{a}$. IN THIS CHAPTER, WE'LL LEARN

- METHOD OF WORK AND ENERGY
- METHOD OF IMPULSE AND MOMENTUM

FOR WORK AND ENERGY, THE INITIAL AND FINAL VELOCITIES ARE RELATED TO THE EXTERNAL FORCES. WE'LL DEFINE:

- WORK OF A FORCE
  - ARBITRARY FORCE
  - CONSTANT FORCE (LIKE FRICTION)
  - FORCE OF GRAVITY
  - ELASTIC FORCE (LIKE A SPRING)
- ENERGY OF A PARTICLE
  - KINETIC
  - POTENTIAL
    * GRAVITIONAL
    * ELASTIC
- POWER = TIME RATE AT WHICH WORK IS DONE.

WE'LL INTEGRATE NEWTON'S SECOND LAW TO DERIVE A RELATIONSHIP BETWEEN THE WORK DONE ON A PARTICLE AND THE CHANGE IN KINETIC ENERGY T:

$$T_1 + U_{1-2} = T_2$$

FOR THE SPECIAL CASE WHERE THERE IS NO FRICTION, THE CONSERVATION OF ENERGY EQUATION STATES THAT THE SUM OF THE KINETIC ENERGY AND POTENTIAL ENERGY REMAINS CONSTANT:

$$T + V = \text{CONSTANT} \quad \text{OR} \quad T_1 + V_1 = T_2 + V_2$$

WHERE V = POTENTIAL ENERGY

FOR IMPULSE AND MOMENTUM, WE'LL AGAIN INTEGRATE NEWTON'S SECOND LAW TO RELATE THE CHANGE OF MOMENTUM OF A PARTICLE TO THE IMPULSE OF AN EXTERNAL FORCE:

$$\Sigma m\vec{v}_1 + \Sigma \overrightarrow{IMP}_{1-2} = \Sigma m\vec{v}_2$$

FOR THE SPECIAL CASE OF IMPACT OF TWO PARTICLES, WE'LL DEFINE THE COEFFICIENT OF RESTITUTION e:

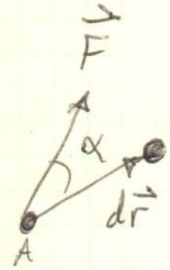
$e = 0$: PERFECTLY PLASTIC

$e = 1$: PERFECTLY ELASTIC

TWO TYPES OF IMPACT WILL BE EXAMINED:

- DIRECT CENTRAL IMPACT
- OBLIQUE CENTRAL IMPACT

METHOD OF WORK AND ENERGY



DIFFERENTIAL WORK OF A FORCE:

$$dU = \vec{F} \cdot d\vec{r} = \text{FORCE} \cdot \text{DISPLACEMENT}$$

INTEGRATE ALONG THE PATH:

$$U_{1-2} = \int \vec{F} \cdot d\vec{r} = \int (F \cos\alpha)\, ds = \int F_t \cdot ds$$

F_t = TANGENTIAL COMPONENT OF FORCE DIRECTED ALONG THE PATH

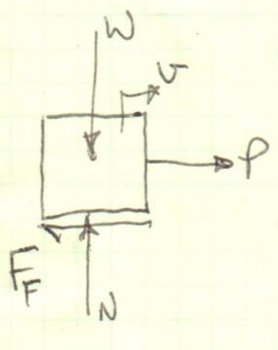
s = ARC LENGTH

FOR A PARTICLE MOVING IN A STRAIGHT LINE WITH A CONSTANT FORCE:

$$U_{1-2} = (F \cos\alpha) \cdot \Delta x$$

CAN BE POSITIVE OR NEGATIVE DEPENDING ON $\cos\alpha$.

FRICTION FORCES ALWAYS OPPOSE MOTION



$$U_{1-2} = (F \cos\alpha) \cdot \Delta x \quad , \alpha = 180^\circ$$

$$= -F_F \cdot \Delta x = -\mu N \cdot \Delta x = -\mu W \cdot \Delta x$$

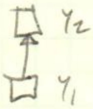
ALWAYS NEGATIVE SINCE $\cos\alpha < 0$

FOR A PARTICLE MOVING IN A GRAVITATIONAL FIELD WHERE THE WEIGHT IS CONSTANT,

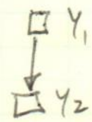
$$\mathcal{U}_{1-2} = W \cdot \Delta y = W(y_1 - y_2) = (V_g)_1 - (V_g)_2$$

WHERE $V_g = W \cdot y$ = POTENTIAL ENERGY OF A GRAVITY FORCE

$\mathcal{U}_{1-2} < 0$ FOR $y_2 > y_1$



$\mathcal{U}_{1-2} > 0$ FOR $y_2 < y_1$



FOR A SPACE VEHICLE, WHERE THE WEIGHT IS NOT CONSTANT,

$$V_g = -\frac{GMm}{r} = -\frac{WR^2}{r}$$

G = UNIV. GRAVITY CONSTANT

M = MASS OF BODY A

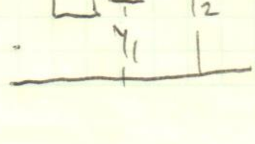
m = MASS OF PARTICLE

r = CENTER-TO-CENTER DISTANCE

W = WEIGHT ON EARTH

R = RADIUS OF EARTH

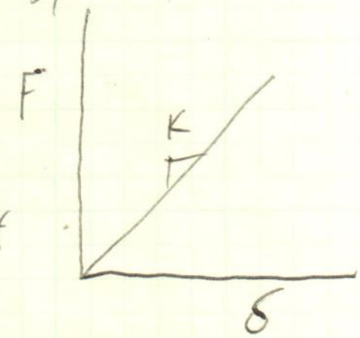
WE'LL ALWAYS BE TRACKING CHANGES IN ELEVATION TO COMPUTE CHANGES IN POTENTIAL ENERGY, SO WE'LL ASSIGN AN ARBITRARY DATUM LINE.



WORK OF A GRAVITY FORCE ONLY DEPENDS ON THE INITIAL AND FINAL ELEVATIONS.

WORK OF AN ELASTIC FORCE ON A PARTICLE BY A SPRING: FORCE IS GIVEN BY

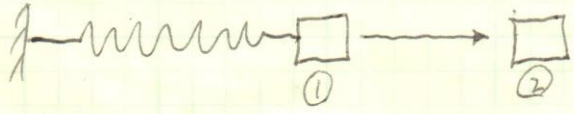
$$F = K\delta = K(l - l_u)$$

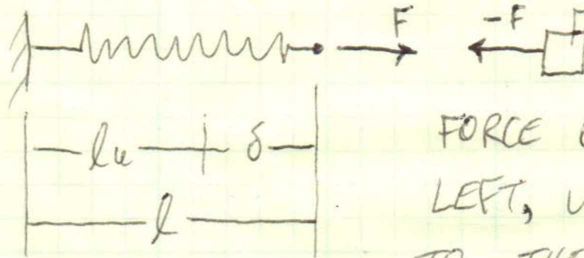


K = SPRING CONSTANT

l = STRETCHED OR COMPRESSED LENGTH

l_u = UNDEFORMED LENGTH

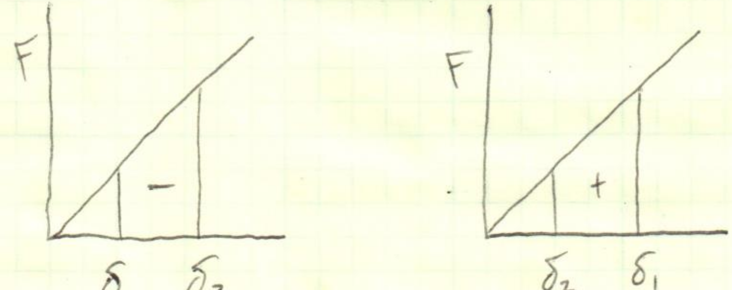




FORCE ON BLOCK IS TO THE LEFT, WHILE THE BLOCK MOVES TO THE RIGHT.

$$U_{1-2} = \int_{\delta_1}^{\delta_2} -K\delta \cdot d\delta = -\tfrac{1}{2}K(\delta_2^2 - \delta_1^2) < 0$$

IF BODY MOVES TO THE LEFT, $0 \le \delta_2 \le \delta_1$ AND $U_{1-2} > 0$



WORK DONE IS AREA UNDER THE CURVE.

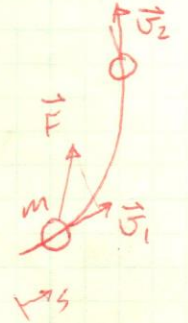
$$U_{1-2} = (V_e)_1 - (V_e)_2 \quad \text{WHERE } V_e = \tfrac{1}{2}K\delta^2$$

V_e = POTENTIAL ENERGY OF AN ELASTIC FORCE.

WORK OF AN ELASTIC FORCE ONLY DEPENDS ON THE INITIAL AND FINAL DEFLECTIONS OF THE SPRING.

KINETIC ENERGY OF A PARTICLE

TANGENTIAL COMPONENTS OF NEWTON'S SECOND LAW:



$$F_t = m a_t = m \frac{dv}{dt} = m \frac{dv}{ds} \cdot \frac{ds}{dt} = m \frac{dv}{ds} \cdot v$$

v = SPEED OF PARTICLE

SEPARATE VARIABLES AND INTEGRATE:

$$F_t \, ds = m v \, dv$$

$$\int_{s_1}^{s_2} F_t \, ds = m \int_{v_1}^{v_2} v \, dv = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$\mathcal{U}_{1-2} = \int_{s_1}^{s_2} F_t \, ds$ = WORK DONE ON A PARTICLE BY EXTERNAL FORCES.

$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = T_2 - T_1$ = CHANGE IN KINETIC ENERGY OF THE PARTICLE.

$\mathcal{U}_{1-2} = T_2 - T_1$ OR $\boxed{T_1 + \mathcal{U}_{1-2} = T_2}$ PRINCIPLE OF WORK AND ENERGY

$$\mathcal{U}_{1-2} = \overset{\text{Gravity}}{(\mathcal{U}_{1-2})_g} + \overset{\text{Elastic}}{(\mathcal{U}_{1-2})_e} + (\mathcal{U}_{1-2})_{FRICTION}$$

$$\mathcal{U}_{1-2} = (V_g)_1 - (V_g)_2 + (V_e)_1 - (V_e)_2 + (\mathcal{U}_{1-2})_F$$

$$T_1 + \left[(V_g)_1 - (V_g)_2 + (V_e)_1 - (V_e)_2 + (U_{1-2})_F\right] = T_2$$

$$T_1 + (V_g)_1 + (V_e)_1 + (U_{1-2})_F = T_2 + (V_g)_2 + (V_e)_2$$

FOR NO FRICTION, $(U_{1-2})_F = 0$, EQUATION REDUCES TO THE CONSERVATION OF ENERGY EQUATION:

$$T_1 + (V_g)_1 + (V_e)_1 = T_2 + (V_g)_2 + (V_e)_2$$

OR $KE + PE = T + V =$ CONSTANT OR

$$T_1 + V_1 = T_2 + V_2$$

METHOD OF IMPULSE AND MOMENTUM

RELATES THE CHANGE IN LINEAR MOMENTUM OF A PARTICLE TO THE IMPULSE OF AN EXTERNAL FORCE. NEWTON'S SECOND LAW:

$$\vec{F} = m\vec{a} = m\,\frac{d\vec{v}}{dt}$$

SEPARATE VARIABLES AND INTEGRATE

$$\int_{t_1}^{t_2} \vec{F}\,dt = m\int_{v_1}^{v_2} d\vec{v} = m\vec{v}_2 - m\vec{v}_1$$

$m\vec{v}_2 - m\vec{v}_1$ = CHANGE IN LINEAR MOMENTUM = VECTOR

$\int_{t_1}^{t_2} \vec{F}\,dt$ = IMPULSE OF FORCE $\vec{F}$ = $\overrightarrow{IMP}_{1-2}$ = VECTOR

$$m\vec{v}_1 + \overrightarrow{IMP}_{1-2} = m\vec{v}_2$$

FOR MULTIPLE EXTERNAL FORCES,

$$m\vec{v}_1 + \sum\overrightarrow{IMP}_{1-2} = m\vec{v}_2$$

FOR MULTIPLE PARTICLES,

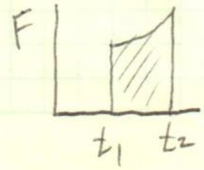
$$\sum m\vec{v}_1 + \sum\overrightarrow{IMP}_{1-2} = \sum m\vec{v}_2$$

IF EXTERNAL FORCES SUM TO ZERO,

$\sum m\vec{v}_1 = \sum m\vec{v}_2$: TOTAL MOMENTUM OF PARTICLES IS CONSERVED.

CALCULATE THE IMPULSE OF A FORCE:

$$\overrightarrow{IMP}_{1-2} = \int_{t_1}^{t_2} \vec{F}(t)\,dt$$



FOR A CONSTANT FORCE,

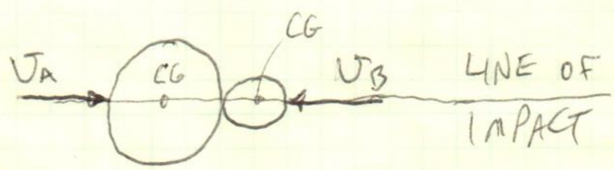
$$\overrightarrow{IMP}_{1-2} = \vec{F}\,(t_2 - t_1) = \vec{F}\cdot\Delta t$$

~~FOR A CONSTANT FORCE,~~

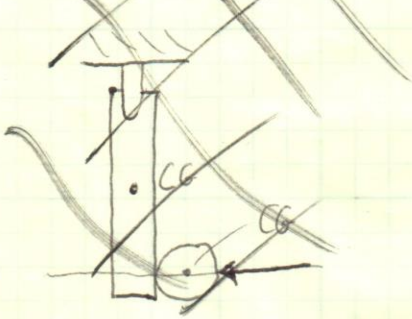
$$\cancel{\overrightarrow{IMP}_{1-2} = \vec{F}(t_2 - t_1) = \vec{F} \cdot \Delta t}$$

IMPACT

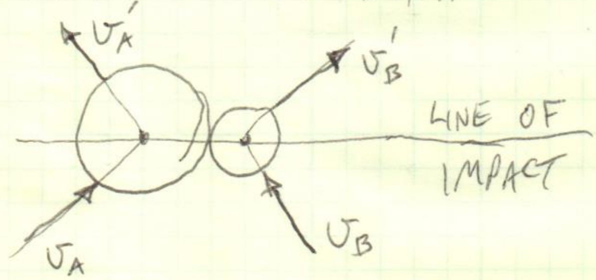
DIRECT CENTRAL IMPACT:



ECCENTRIC IMPACT:



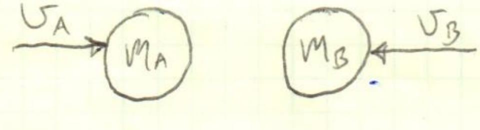
OBLIQUE CENTRAL IMPACT:



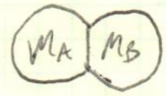
DIRECT CENTRAL IMPACT: NO EXTERNAL FORCES, SO THE TOTAL MOMENTUM OF THE PARTICLES IS CONSERVED.

$$\Sigma m\vec{v}_1 = \Sigma m\vec{v}_2$$

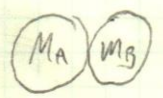
$$\underbrace{m_A \vec{v}_A + m_B \vec{v}_B}_{\text{BEFORE IMPACT}} = \underbrace{m_A v_A' + m_B v_B'}_{\text{AFTER IMPACT}}$$



BEFORE IMPACT



DURING IMPACT: MAXIMUM DEFORMATION



AFTER: PERIOD OF RESTITUTION

COEFFICIENT OF RESTITUTION: RELATIVE VELOCITIES BEFORE AND AFTER IMPACT

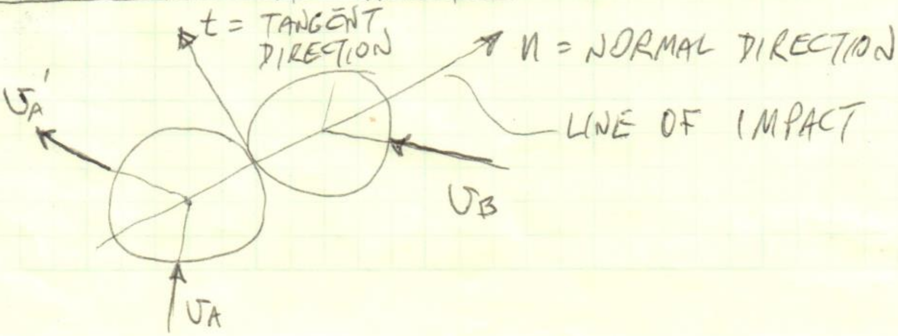
$$e = \frac{(v_B' - v_A')^{\text{AFTER}}}{(v_A - v_B)_{\text{BEFORE}}} \quad \text{OR} \quad (v_B' - v_A') = e(v_A - v_B)$$

$0 \le e \le 1$

FOR $e = 0$, PERFECTLY PLASTIC IMPACT: PARTICLES STAY TOGETHER AFTER IMPACT, AND MAXIMUM ENERGY IS LOST DURING DEFORMATION

FOR $e = 1$, PERFECTLY ELASTIC IMPACT: NO ENERGY IS LOST, AND MOMENTUM OF BOTH PARTICLES IS CONSERVED.

<u>OBLIQUE CENTRAL IMPACT</u>



TANGENT DIRECTION: t - COMPONENT OF MOMENTUM OF EACH PARTICLE IS CONSERVED:

$$(v_A)_t = (v_A')_t \quad \underline{\text{AND}} \quad (v_B)_t = (v_B')_t$$

NORMAL DIRECTION: n - COMPONENT OF TOTAL MOMENTUM IS CONSERVED.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v_A')_n + m_B(v_B')_n$$

BEFORE/AFTER NORMAL RELATIVE VELOCITIES ARE RELATED BY COEFFICIENT OF RESTITUTION:

$$(v_B')_n - (v_A')_n = e\left[(v_A)_n - (v_B)_n\right]$$

SUMMARY OF THREE KINETICS SOLUTION METHODS:

- DIRECT APPLICATION OF NEWTON'S 2ND LAW ($\vec{F} = m\vec{a}$): MOST CUMBERSOME METHOD, BUT IT IS OFTEN USED TO SUPPLEMENT THE WORK AND ENERGY METHOD.

- WORK AND ENERGY: SIMPLIFIES THE SOLUTION OF PROBLEMS DEALING WITH FORCES, DISPLACEMENTS AND VELOCITIES, SINCE IT DOES NOT REQUIRE DETERMINATION OF ACCELERATIONS. CAN'T BE USED FOR IMPACT PROBLEMS DUE TO THE LOSS OF MECHANICAL ENERGY

- IMPULSE AND MOMENTUM: EFFECTIVE FOR ANALYZING IMPULSIVE MOTION, WHERE LARGE FORCES ACT OVER VERY SHORT TIME INTERVALS. THIS IS THE ONLY SOLUTION METHOD FOR IMPACT PROBCEMS.