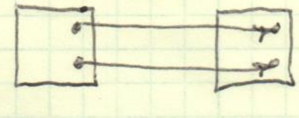
RIGID BODY MOTION (PLANAR MOTION)

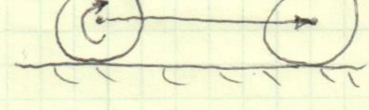
RECTILINEAR TRANSLATION:



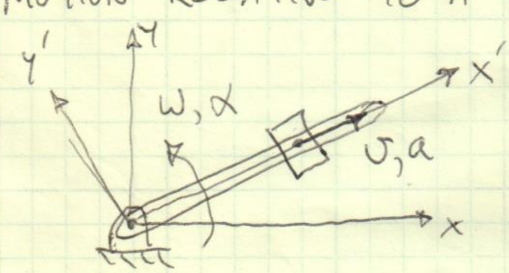
CURVILINEAR TRANSLATION:

ROTATION ABOUT A FIXED AXIS:

GENERAL PLANE MOTION:

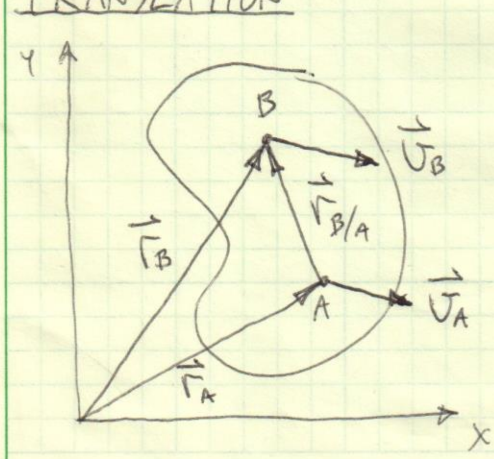


MOTION RELATIVE TO A ROTATING REFERENCE FRAME:



OBJECTIVE: DETERMINE THE POSITION, VELOCITY, ACCELERATION, ANGULAR VELOCITY AND ANGULAR ACCELERATION OF ANY POINT ON A RIGID BODY.

TRANSLATION



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

DIFFERENTIATE W.R.T. TIME:

$$\frac{d}{dt}\left(\vec{r}_B\right) = \frac{d}{dt}\left(\vec{r}_A\right) + \cancelto{0}{\frac{d}{dt}\left(\vec{r}_{B/A}\right)}$$

$$\vec{v}_B = \vec{v}_A$$

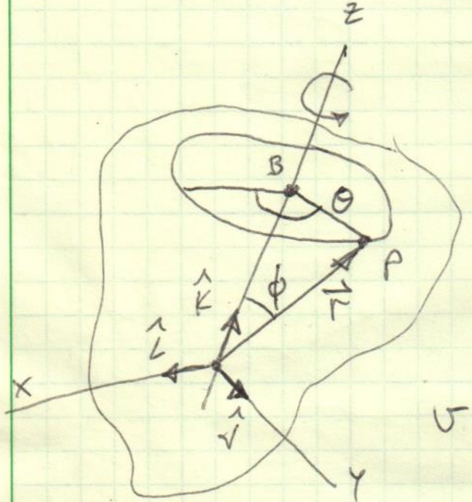
$$\cancel{a_B = a_A}$$

$$\frac{d}{dt}\left(\vec{v}_B\right) = \frac{d}{dt}\left(\vec{v}_A\right)$$

$$\vec{a}_B = \vec{a}_A$$

FOR A BODY IN TRANSLATION, ALL POINTS HAVE THE SAME VELOCITY AND ACCELERATION.

ROTATION ABOUT A FIXED AXIS



ARC-LENGTH TRAVELED BY P:

$$s = R\theta = (r \sin\phi)\cdot\theta$$

VELOCITY OF P:

$$v = \frac{ds}{dt} = (r\sin\phi)\frac{d\theta}{dt}$$

$$v = (r\sin\phi)\dot{\theta} = (r\sin\phi)\,\omega$$

WHERE ω IS THE ANGULAR VELOCITY. IN VECTOR NOTATION,

$$\boxed{\vec{v} = \frac{d}{dt}\left(\vec{r}\right) = \vec{\omega}\times\vec{r} = \omega\hat{k}\times\vec{r}}$$ (FOR PLANAR MOTION)

THE ACCELERATION OF POINT P IS (PRODUCT RULE)

$$\vec{a} = \frac{d}{dt}\left(\vec{v}\right) = \frac{d}{dt}\left(\vec{\omega}\times\vec{r}\right) = \frac{d}{dt}\left(\vec{\omega}\right)\times\vec{r} + \vec{\omega}\times\frac{d}{dt}\left(\vec{r}\right)$$

$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{v} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$

WHERE $\vec{\alpha} = \alpha\hat{k}$ IS THE ANGULAR ACCELERATION.

LET $\vec{\omega} = \omega\hat{k}$ AND $\vec{r} = (x)\hat{i} + (y)\hat{j}$ (PLANAR MOTION)

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ x & y & 0 \end{vmatrix}$$

$$= [0 - (\omega)(y)]\hat{i} - [0 - (\omega)(x)]\hat{j}$$

$$= (-y\omega)\hat{i} + (x\omega)\hat{j}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ -y\omega & x\omega & 0 \end{vmatrix}$$

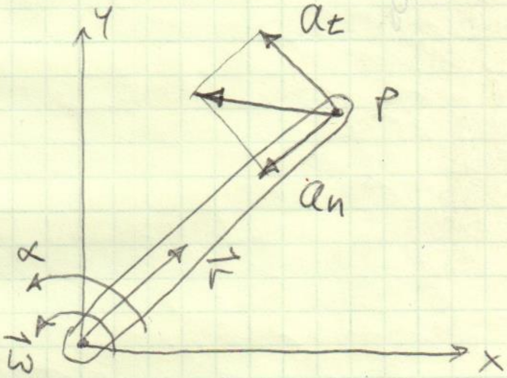
$$= [0 - (\omega)(x\omega)]\hat{i} - [0 - (\omega)(-y\omega)]\hat{j}$$

$$= (-x\omega^2)\hat{i} + (-y\omega^2)\hat{j}$$

$$= -\omega^2[(x)\hat{i} + (y)\hat{j}]$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2 \vec{r}$$

$$\boxed{\vec{a} = \alpha\hat{k} \times \vec{r} - \omega^2 \vec{r}}$$



$\vec{a}_t = \alpha\hat{k} \times \vec{r}$ = TANGENTIAL ACCELERATION

$\vec{a}_n = -\omega^2 \vec{r}$ = NORMAL ACCELERATION

~~GENERAL PLANE MOTION~~

~~G.P.M. = TRANSLATION + ROTATION~~

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EQUATIONS FOR ROTATION OF A RIGID BODY ABOUT A FIXED AXIS:

$$\omega = \frac{d\theta}{dt} = \text{ANGULAR VELOCITY}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \text{ANGULAR ACCELERATION}$$

$$dt = \frac{d\theta}{\omega}, \quad \alpha = \frac{d\omega}{\left(\frac{d\theta}{\omega}\right)} = \omega\frac{d\omega}{d\theta}$$

FOR UNIFORM ROTATION ($\alpha = 0$, ω = CONSTANT)

$$\theta = \theta_0 + \omega t \qquad (\text{SIMILAR TO } x = x_0 + v t)$$

FOR UNIFORMLY ACCELERATED ROTATION (α = CONSTANT)

$$\omega = \omega_0 + \alpha t \qquad (v = v_0 + at)$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad (x = x_0 + v_0 t + \tfrac{1}{2}at^2)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \qquad [v^2 = v_0^2 + 2a(x - x_0)]$$

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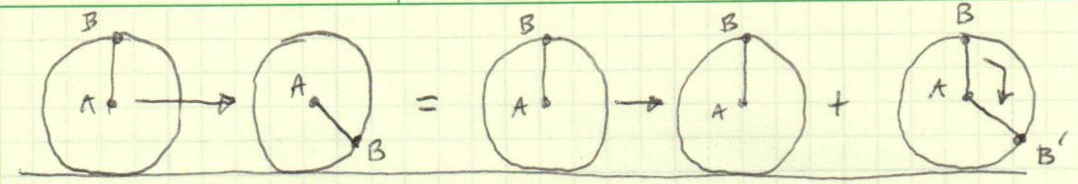
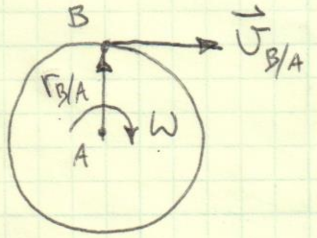
GENERAL PLANE MOTION: TRANSLATION + ROTATION



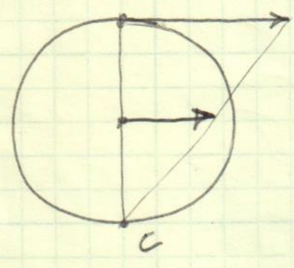
VELOCITY: TRANSLATION + ROTATION ABOUT A

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \omega\hat{k} \times \vec{r}_{B/A}$$

$\vec{v}_{B/A} = \omega\hat{k} \times \vec{r}_{B/A}$ = RELATIVE VELOCITY OF B W.R.T. A



INSTANTANEOUS CENTER OF ROTATION: THE POINT ABOUT WHICH YOU CAN ASSUME A BODY IS ROTATING AT A GIVEN INSTANT. THIS CAN BE USED TO FIND ω OR VELOCITIES ON THE BODY.



ACCELERATION: $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}_{B/A} = \alpha\hat{k} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$ = RELATIVE ACCELERATION OF B W.R.T. A

$$\vec{a}_B = \vec{a}_A + \alpha\hat{k} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$$

~~MOTION RELATIVE TO A ROTATING REFERENCE FRAME~~

~~SLIDING CONNECTIONS CAN BE ANALYZED USING A ROTATING REFERENCE FRAME.~~