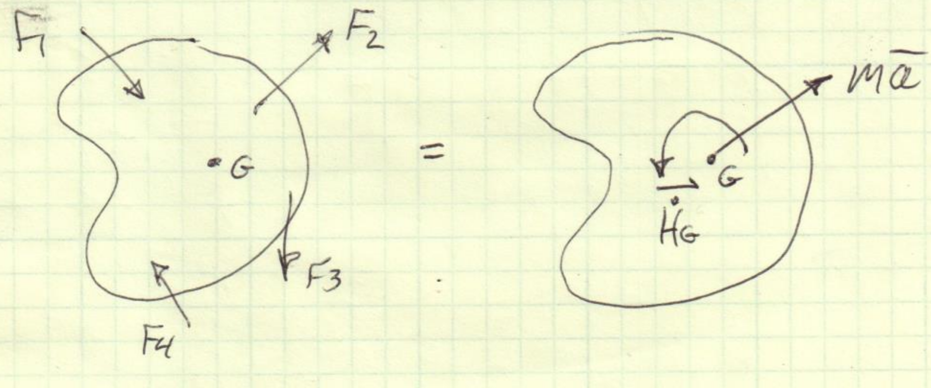
PREVIOUSLY, WE STUDIED THE MOTION OF PARTICLES, WHERE THE MASS AND ALL OF THE FORCES WERE CONCENTRATED AT A SINGLE POINT. IN THIS CHAPTER, WE'LL ACCOUNT FOR THE SHAPE OF THE BODY AND THE LOCATION OF EACH POINT OF APPLICATION OF EACH FORCE. THIS ANALYSIS IS RESTRICTED TO PLANAR SLABS OR BODIES THAT ARE SYMMETRICAL TO THE PLANE OF INTEREST.

RECALLING CHAP. 14, FOR A SYSTEM OF PARTICLES:

$$\Sigma \vec{F} = m\bar{a} \quad , \quad \Sigma \vec{M}_G = \dot{\vec{H}}_G$$

$\bar{a}$ = ACCELERATION OF THE MASS CENTER G

$\dot{\vec{H}}_G$ = RATE OF CHANGE OF THE ANGULAR MOMENTUM ABOUT MASS CENTER G

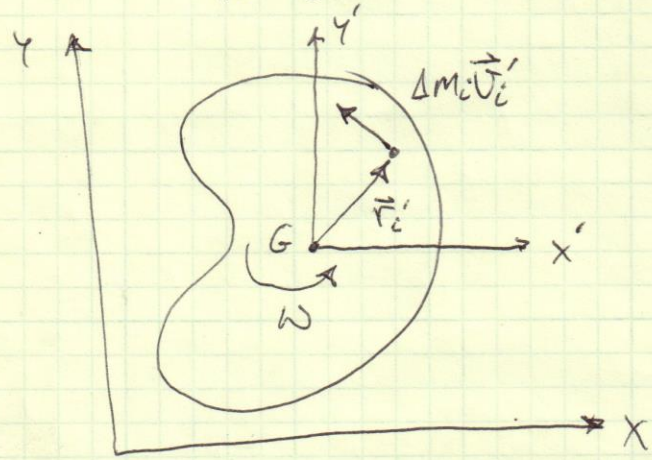


THE ANGULAR MOMENTUM OF THE RIGID BODY IS GIVEN BY EQN. 14-24:

$$\vec{H}_G = \sum_{i=1}^{n} \left(\vec{r}_i' \times \Delta m_i \vec{v}_i' \right)$$

$\vec{r}_i'$ = POSITION VECTOR FROM G TO MASS Δm_i

$\Delta m_i \vec{v}_i'$ = LINEAR MOMENTUM OF MASS Δm_i RELATIVE TO G



$$\vec{v}_i' = \vec{\omega} \times \vec{r}_i'$$

$\vec{\omega}$ = ANGULAR VELOCITY OF THE SLAB

$$\vec{H}_G = \sum_{i=1}^{n} \left[\vec{r}_i' \times \left(\vec{\omega} \times \vec{r}_i' \right) \Delta m_i \right] = \left[\omega \cdot \Sigma \left(\vec{r}_i'^{\,2} \Delta m_i \right) \right] \hat{k}$$

(PERPENDICULAR TO THE SLAB)

FROM STATICS,

$\bar{I} = \Sigma \left(r_i'^{\,2} \Delta m_i \right)$ = MASS MOMENT OF INERTIA OF THE SLAB ABOUT THE PERPENDICULAR CENTROIDAL AXIS.

$\vec{H}_G = \bar{I} \vec{\omega}$ ANGULAR MOMENTUM OF THE SLAB

DIFF. W.R.T. TIME GIVES

$$\dot{\vec{H}}_G = \bar{I}\,\dot{\vec{\omega}} = \bar{I}\,\vec{\alpha}$$

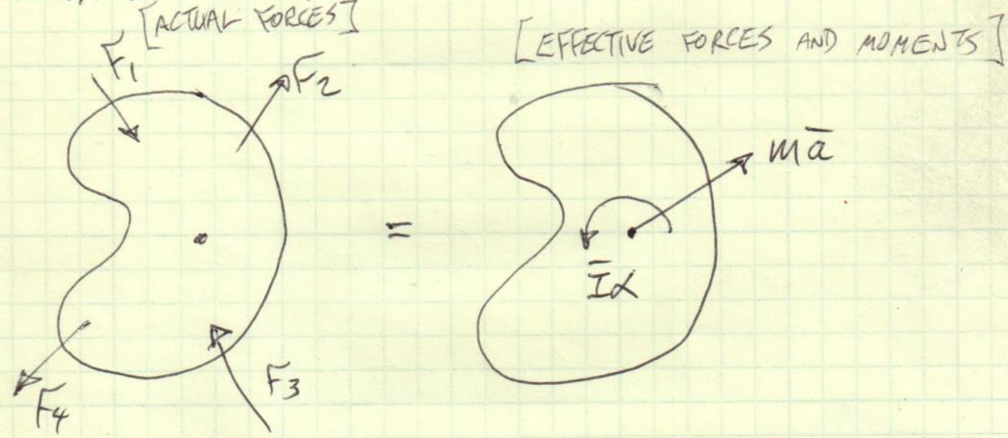
THEREFORE,

$$\boxed{\Sigma \vec{M}_G = \bar{I}\,\vec{\alpha}}$$

FOR TWO-DIMENSIONAL PLANAR MOTION, THE FUNDAMENTAL EQUATIONS OF MOTION ARE

$$\Sigma F_x = m\bar{a}_x \;,\; \Sigma F_y = m\bar{a}_y \;,\; \Sigma M_G = \bar{I}\alpha$$

THE MOTION OF THE SLAB IS COMPLETELY DEFINED BY THE FORCE RESULTANT AND THE MOMENT RESULTANT ABOUT G OF THE EXTERNAL FORCES.



FOR TRANSLATION ONLY, $\Sigma \vec{M}_G = 0$, $\Sigma \vec{F} = m\bar{a}$

FOR CENTROIDAL ROTATION ONLY, $\Sigma \vec{F} = 0$, $\Sigma \vec{M}_G = \bar{I}\vec{\alpha}$

FOR ROLLING MOTION WITHOUT SLIDING,

$\bar{a} = r\alpha$, WHERE $F_F \le \mu_s N$

FOR SLIDING IMPENDING, $F_F = \mu_s N$, $\bar{a} = r\alpha$

FOR ROLLING WITH SLIDING,

$\bar{a} \neq r\alpha$, $F_F = \mu_k N$

TO SOLVE THIS TYPE OF PROBLEM, FIRST ASSUME THE DISK ROLLS WITHOUT SLIDING.

IF $F_F \leq \mu_s N$, ASSUMPTION WAS CORRECT

IF $F_F > \mu_s N$, DISK SLIDES : REDO PROBLEM