ME 1020 Engineering Programming with MATLAB

Handout 04b

Homework 4b Assignment: 4.21, 4.26, 4.31, 4.34, 4.36

Section 4.5

21. Use a `for` loop to plot the function given in Problem 16 over the interval $-2 \leq x \leq 6$. Properly label the plot. The variable y represents height in kilometers, and the variable x represents time in seconds.

22. Use a `for` loop to determine the sum of the first 10 terms in the series $5k^3$, $k = 1, 2, 3, \ldots, 10$.

23. The (x, y) coordinates of a certain object as a function of time t are given by

$$x(t) = 5t - 10 \qquad y(t) = 25t^2 - 120t + 144$$

for $0 \leq t \leq 4$. Write a program to determine the time at which the object is the closest to the origin at (0, 0). Determine also the minimum distance. Do this in two ways:

a. By using a `for` loop.

b. By not using a `for` loop.

24. Consider the array **A**.

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & -4 \\ -8 & -1 & 33 \\ -17 & 6 & -9 \end{bmatrix}$$

Write a program that computes the array **B** by computing the natural logarithm of all the elements of **A** whose value is no less than 1, and adding 20 to each element that is equal to or greater than 1. Do this in two ways:

a. By using a `for` loop with conditional statements.

b. By using a logical array as a mask.

25. We want to analyze the mass-spring system discussed in Problem 20 for the case in which the weight W is dropped onto the platform attached to the center spring. If the weight is dropped from a height h above the platform, we can find the maximum spring compression x by equating the weight's gravitational potential energy $W(h + x)$ with the potential energy stored in the springs. Thus

$$W(h + x) = \frac{1}{2}k_1x^2 \qquad \text{if } x < d$$

which can be solved for x as

$$x = \frac{W \pm \sqrt{W^2 + 2k_1Wh}}{k_1} \qquad \text{if } x < d$$

and

$$W(h + x) = \frac{1}{2}k_1x^2 + \frac{1}{2}(2k_2)(x - d)^2 \qquad \text{if } x \geq d$$

which gives the following quadratic equation to solve for x:

$$(k_1 + 2k_2)x^2 - (4k_2d + 2W)x + 2k_2d^2 - 2\,Wh = 0 \qquad \text{if } x \geq d$$

a. Create a function file that computes the maximum compression x due to the falling weight. The function's input parameters are k_1, k_2, d, W, and h. Test your function for the following two cases, using the values $k_1 = 10^4$ N/m; $k_2 = 1.5 \times 10^4$ N/m; and $d = 0.1$ m.

$$W = 100 \text{ N} \qquad h = 0.5 \text{ m}$$

$$W = 2000 \text{ N} \qquad h = 0.5 \text{ m}$$

b. Use your function file to generate a plot of x versus h for $0 \leq h \leq 2$ m. Use $W = 100$ N and the preceding values for k_1, k_2, and d.

26. Electrical resistors are said to be connected "in series" if the same current passes through each and "in parallel" if the same voltage is applied across each. If in series, they are equivalent to a single resistor whose resistance is given by

$$R = R_1 + R_2 + R_3 + \cdots + R_n$$

If in parallel, their equivalent resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Write an M-file that prompts the user for the type of connection (series or parallel) and the number of resistors n and then computes the equivalent resistance.

27. *a.* An *ideal* diode blocks the flow of current in the direction opposite that of the diode's arrow symbol. It can be used to make a *half-wave rectifier* as shown in Figure P27a. For the ideal diode, the voltage v_L across the load R_L is given by

$$v_L = \begin{cases} v_S & \text{if } v_S > 0 \\ 0 & \text{if } v_S \leq 0 \end{cases}$$

Suppose the supply voltage is

$$v_S(t) = 3e^{-t/3}\sin(\pi t) \qquad \text{V}$$

where time t is in seconds. Write a MATLAB program to plot the voltage v_L versus t for $0 \leq t \leq 10$.

b. A more accurate model of the diode's behavior is given by the *offset diode* model, which accounts for the offset voltage inherent in semiconductor diodes. The offset model contains an ideal diode and a battery whose voltage equals the offset voltage (which is approximately

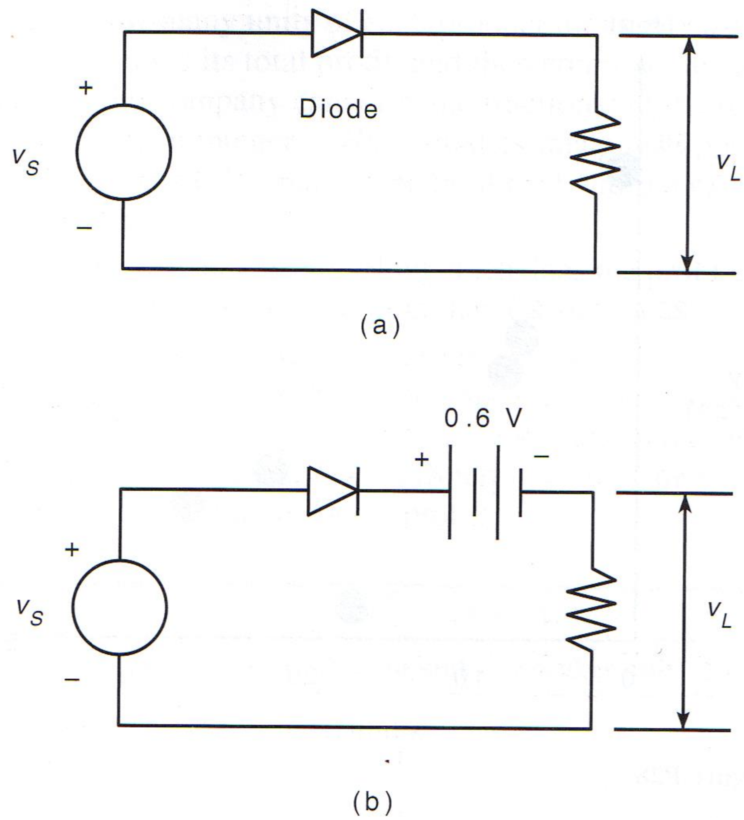

Figure P27

0.6 V for silicon diodes) [Rizzoni, 2007]. The half-wave rectifier using this model is shown in Figure P27b. For this circuit,

$$v_L = \begin{cases} v_S - 0.6 & \text{if } v_S > 0.6 \\ 0 & \text{if } v_S \le 0.6 \end{cases}$$

Using the same supply voltage given in part a, plot the voltage v_L versus t for $0 \le t \le 10$; then compare the results with the plot obtained in part a.

28.* A company wants to locate a distribution center that will serve six of its major customers in a 30 × 30 mi area. The locations of the customers relative to the southwest corner of the area are given in the following table in terms of (x, y) coordinates (the x direction is east; the y direction is north) (see Figure P28). Also given is the volume in tons per week that must be delivered from the distribution center to each customer. The weekly delivery cost c_i for customer i depends on the volume V_i and the distance d_i from the distribution center. For simplicity we will assume that this distance is the straight-line distance. (This assumes that the road network is dense.) The weekly cost is given by $c_i = 0.5 d_i V_i$, $i = 1, \ldots, 6$. Find the location of the distribution center (to the nearest mile) that minimizes the total weekly cost to service all six customers.

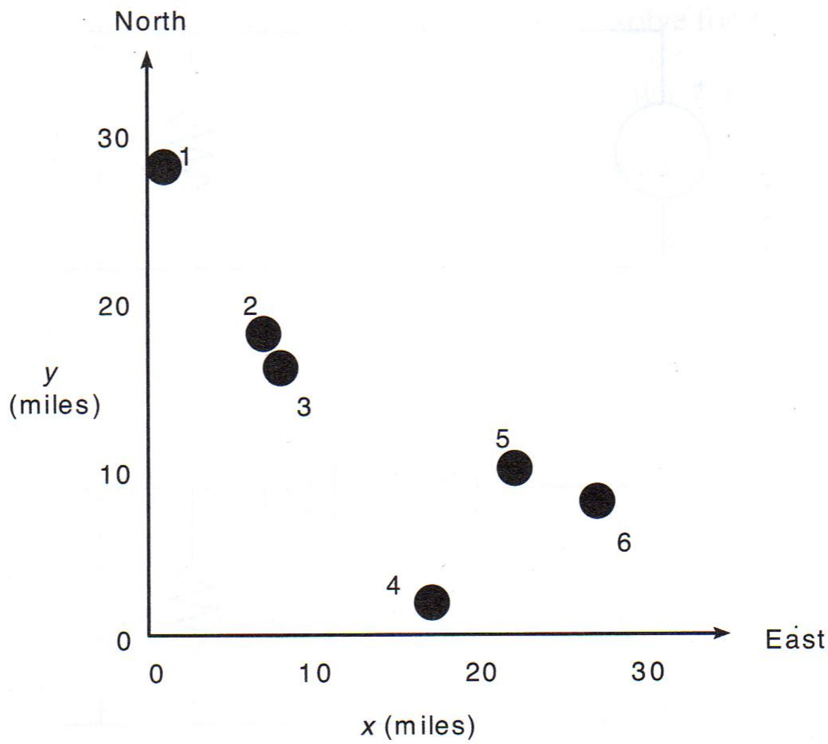

Figure P28

Customer	x location (mi)	y location (mi)	Volume (tons/week)
1	1	28	3
2	7	18	7
3	8	16	4
4	17	2	5
5	22	10	2
6	27	8	6

29. A company has the choice of producing up to four different products with its machinery, which consists of lathes, grinders, and milling machines. The number of hours on each machine required to produce a product is given in the following table, along with the number of hours available per week on each type of machine. Assume that the company can sell everything it produces. The profit per item for each product appears in the last line of the table.

	Product				
	1	2	3	4	Hours available
Hours required					
Lathe	1	2	0.5	3	40
Grinder	0	2	4	1	30
Milling	3	1	5	2	45
Unit profit ($)	100	150	90	120	

a. Determine how many units of each product the company should make to maximize its total profit, and then compute this profit. Remember, the company cannot make fractional units, so your answer must be in integers. (*Hint:* First estimate the upper limits on the number of products that can be produced without exceeding the available capacity.)

b. How sensitive is your answer? How much does the profit decrease if you make one more or one less item than the optimum?

30. A certain company makes televisions, stereo units, and speakers. Its parts inventory includes chassis, picture tubes, speaker cones, power supplies, and electronics. The inventory, required components, and profit for each product appear in the following table. Determine how many of each product to make in order to maximize the profit.

	Product			
	Television	**Stereo unit**	**Speaker unit**	**Inventory**
Requirements				
Chassis	1	1	0	450
Picture tube	1	0	0	250
Speaker cone	2	2	1	800
Power supply	1	1	0	450
Electronics	2	2	1	600
Unit profit ($)	80	50	40	

Section 4.6

31. Plot the function $y = 10(1 - e^{-x/4})$ over the interval $0 \leq x \leq x_{max}$, using a `while` loop to determine the value of x_{max} such that $y(x_{max}) = 9.8$. Properly label the plot. The variable y represents force in newtons, and the variable x represents time in seconds.

32. Use a `while` loop to determine how many terms in the series 2^k, $k = 1, 2, 3, \ldots$, are required for the sum of the terms to exceed 2000. What is the sum for this number of terms?

33. One bank pays 5.5 percent annual interest, while a second bank pays 4.5 percent annual interest. Determine how much longer it will take to accumulate at least $50 000 in the second bank account if you deposit $1000 initially and $1000 at the end of each year.

34.* Use a loop in MATLAB to determine how long it will take to accumulate $1 000 000 in a bank account if you deposit $10 000 initially and $10 000 at the end of each year; the account pays 6 percent annual interest.

35. A weight W is supported by two cables anchored a distance D apart (see Figure P35). The cable length L_{AB} is given, but the length L_{AC} is to be

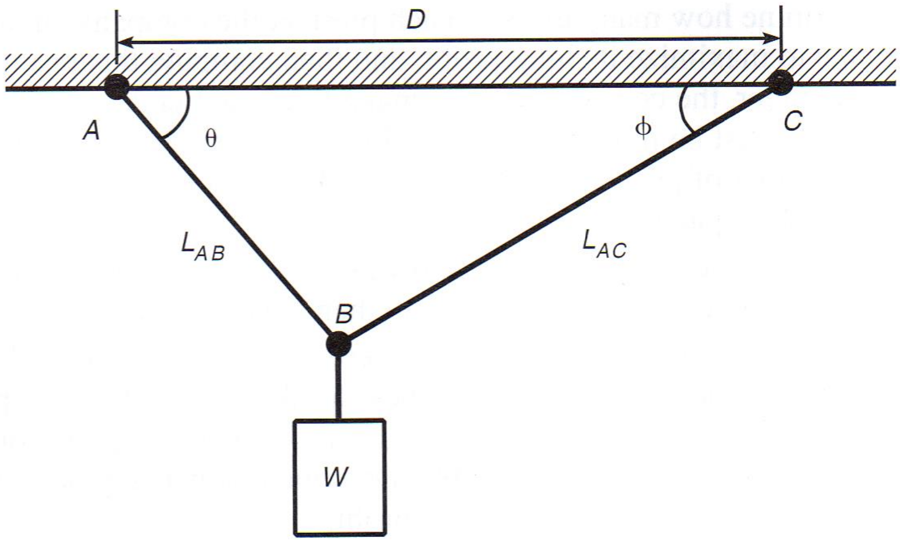

Figure P35

selected. Each cable can support a maximum tension force equal to W. For the weight to remain stationary, the total horizontal force and total vertical force must each be zero. This principle gives the equations

$$-T_{AB}\cos\theta + T_{AC}\cos\phi = 0$$

$$T_{AB}\sin\theta + T_{AC}\sin\phi = W$$

We can solve these equations for the tension forces T_{AB} and T_{AC} if we know the angles θ and ϕ. From the law of cosines

$$\theta = \cos^{-1}\left(\frac{D^2 + L_{AB}^2 - L_{AC}^2}{2DL_{AB}}\right)$$

From the law of sines

$$\phi = \sin^{-1}\left(\frac{L_{AB}\sin\theta}{L_{AC}}\right)$$

For the given values $D = 6$ ft, $L_{AB} = 3$ ft, and $W = 2000$ lb, use a loop in MATLAB to find $L_{AC\min}$, the shortest length L_{AC} we can use without T_{AB} or T_{AC} exceeding 2000 lb. Note that the largest L_{AC} can be is 6.7 ft (which corresponds to $\theta = 90°$). Plot the tension forces T_{AB} and T_{AC} on the same graph versus L_{AC} for $L_{AC\min} \le L_{AC} \le 6.7$.

36.* In the structure in Figure P36a, six wires support three beams. Wires 1 and 2 can support no more than 1200 N each, wires 3 and 4 can support no more than 400 N each, and wires 5 and 6 can support no more than 200 N each. Three equal weights W are attached at the points shown. Assuming that the structure is stationary and that the weights of the wires and the beams are very small compared to W, the principles of statics applied to a particular beam state that the sum of vertical forces is zero and

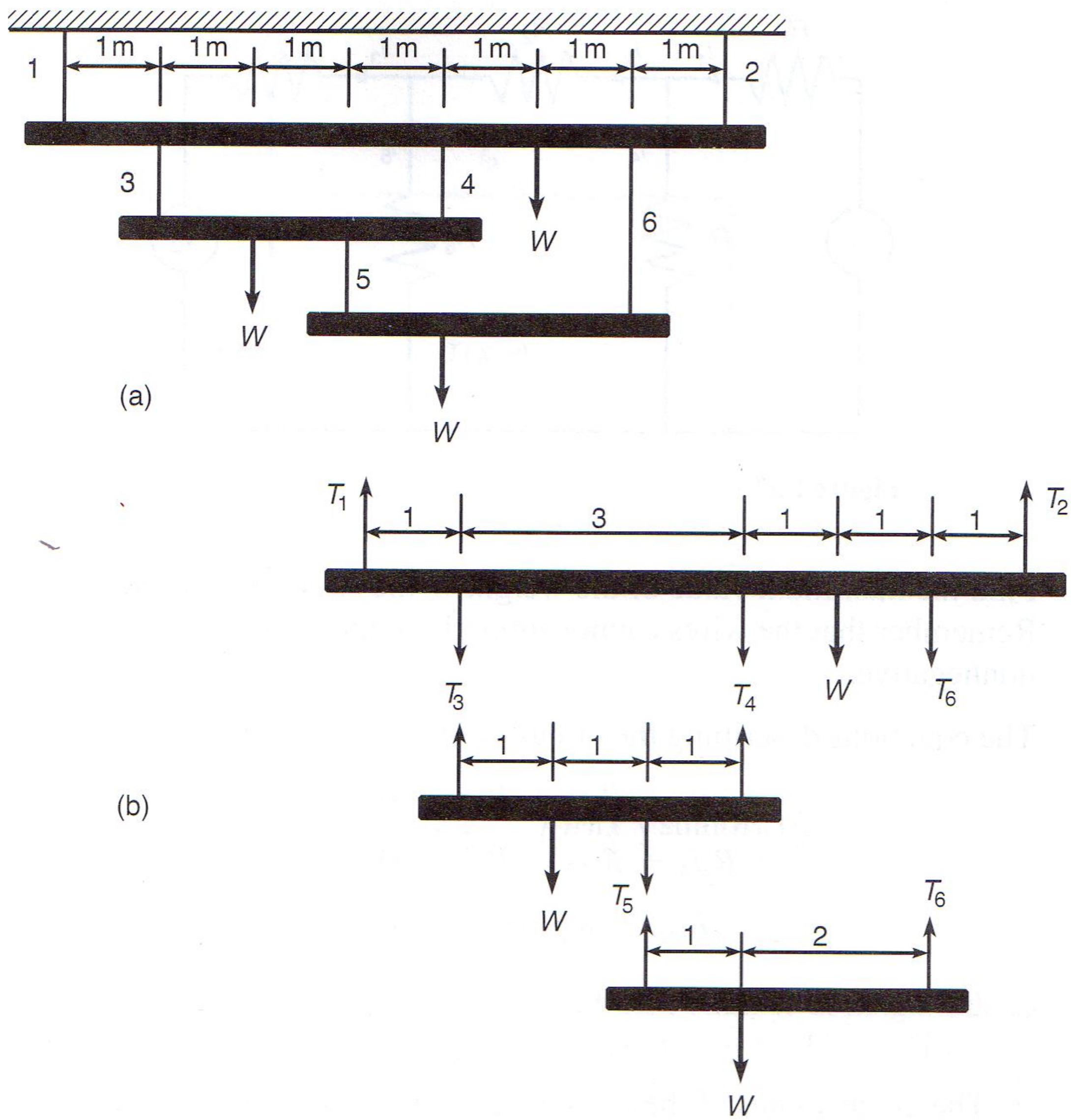

Figure P36

that the sum of moments about any point is also zero. Applying these principles to each beam using the free-body diagrams shown in Figure P36b, we obtain the following equations. Let the tension force in wire i be T_i. For beam 1

$$T_1 + T_2 = T_3 + T_4 + W + T_6$$

$$-T_3 - 4T_4 - 5W - 6T_6 + 7T_2 = 0$$

For beam 2

$$T_3 + T_4 = W + T_5$$

$$-W - 2T_5 + 3T_4 = 0$$

For beam 3

$$T_5 + T_6 = W$$

$$-W + 3T_6 = 0$$

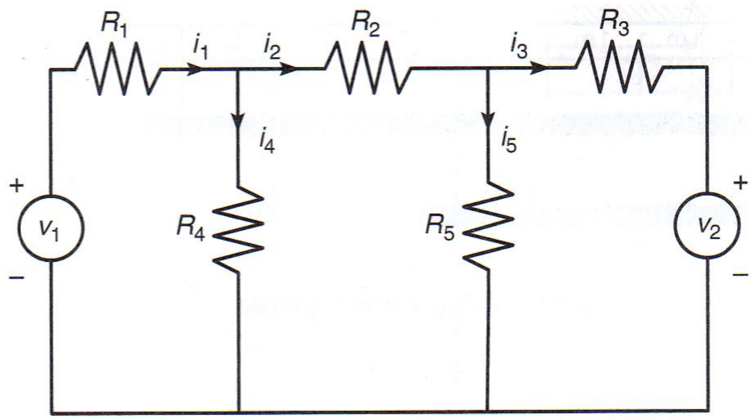

Figure P37

Find the maximum value of the weight W the structure can support. Remember that the wires cannot support compression, so T_i must be nonnegative.

37. The equations describing the circuit shown in Figure P37 are

$$-v_1 + R_1 i_1 + R_4 i_4 = 0$$
$$-R_4 i_4 + R_2 i_2 + R_5 i_5 = 0$$
$$-R_5 i_5 + R_3 i_3 + v_2 = 0$$
$$i_1 = i_2 + i_4$$
$$i_2 = i_3 + i_5$$

a. The given values of the resistances and the voltage v_1 are $R_1 = 5$, $R_2 = 100$, $R_3 = 200$, $R_4 = 150$, $R_5 = 250$ kΩ, and $v_1 = 100$ V. (Note that 1 kΩ = 1000 Ω.) Suppose that each resistance is rated to carry a current of no more than 1 mA (= 0.001 A). Determine the allowable range of positive values for the voltage v_2.

b. Suppose we want to investigate how the resistance R_3 limits the allowable range for v_2. Obtain a plot of the allowable limit on v_2 as a function of R_3 for $150 \le R_3 \le 250$ kΩ.

38. Many applications require us to know the temperature distribution in an object. For example, this information is important for controlling the material properties, such as hardness, when cooling an object formed from molten metal. In a heat-transfer course, the following description of the temperature distribution in a flat, rectangular metal plate is often derived. The temperature is held constant at T_1 on three sides and at T_2 on the fourth side (see Figure P38). The temperature $T(x, y)$ as a function of the xy coordinates shown is given by

$$T(x, y) = (T_2 - T_1)w(x, y) + T_1$$

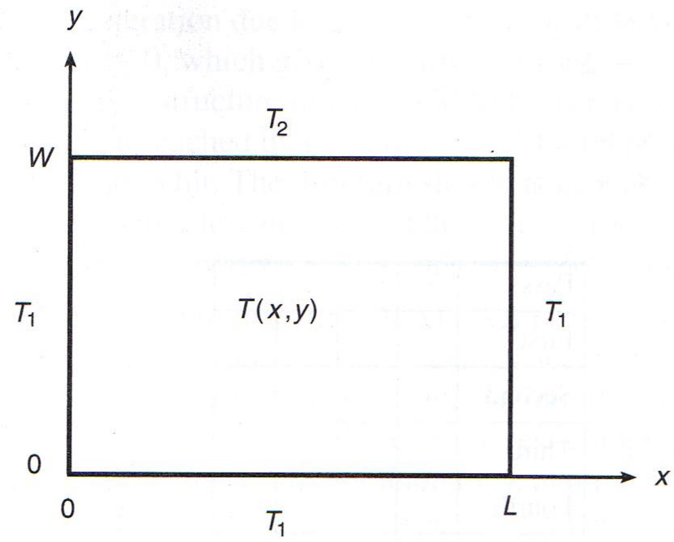

Figure P38

where

$$w(x,y) = \frac{2}{\pi}\sum_{n \text{ odd}}^{\infty} \frac{2}{n} \sin\left(\frac{n\pi x}{L}\right)\frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$$

Use the following data: $T_1 = 70°\text{F}$, $T_2 = 200°\text{F}$, and $W = L = 2$ ft.

a. The terms in the preceding series become smaller in magnitude as n increases. Write a MATLAB program to verify this fact for $n = 1, \ldots, 19$ for the center of the plate ($x = y = 1$).

b. Using $x = y = 1$, write a MATLAB program to determine how many terms are required in the series to produce a temperature calculation that is accurate to within 1 percent. (That is, for what value of n will the addition of the next term in the series produce a change in T of less than 1 percent?) Use your physical insight to determine whether this answer gives the correct temperature at the center of the plate.

c. Modify the program from part *b* to compute the temperatures in the plate; use a spacing of 0.2 for both x and y.

39. Consider the following script file. Fill in the lines of the following table with the values that would be displayed immediately after the `while` statement if you ran the script file. Write in the values the variables have each time the `while` statement is executed. You might need more or fewer lines in the table. Then type in the file, and run it to check your answers.

```
k = 1;b = -2;x = -1;y = -2;
while k <= 3
    k, b, x, y
    y = x^2 - 3;
```

```
    if y < b
       b = y;
    end
    x = x + 1;
    k = k + 1;
end
```

Pass	k	b	x	y
First				
Second				
Third				
Fourth				
Fifth				

40. Assume that the human player makes the first move against the computer in a game of Tic-Tac-Toe, which has a 3×3 grid. Write a MATLAB function that lets the computer respond to that move. The function's input argument should be the cell location of the human player's move. The function's output should be the cell location of the computer's first move. Label the cells as 1, 2, 3 across the top row; 4, 5, 6 across the middle row; and 7, 8, 9 across the bottom row.

Section 4.7

41. The following table gives the approximate values of the static coefficient of friction μ for various materials.

Materials	μ
Metal on metal	0.20
Wood on wood	0.35
Metal on wood	0.40
Rubber on concrete	0.70

To start a weight W moving on a horizontal surface, you must push with a force F, where $F = \mu W$. Write a MATLAB program that uses the `switch` structure to compute the force F. The program should accept as input the value of W and the type of materials.

42. The height and speed of a projectile (such as a thrown ball) launched with a speed of v_0 at an angle A to the horizontal are given by

$$h(t) = v_0 t \sin A - 0.5gt^2$$

$$v(t) = \sqrt{v_0^2 - 2v_0 gt \sin A + g^2t^2}$$

where g is the acceleration due to gravity. The projectile will strike the ground when $h(t) = 0$, which gives the time to hit $t_{hit} = 2(v_0/g) \sin A$.

Use the `switch` structure to write a MATLAB program to compute the maximum height reached by the projectile, the total horizontal distance traveled, or the time to hit. The program should accept as input the user's choice of which quantity to compute and the values of v_0, A, and g. Test the program for the case where $v_0 = 40$ m/s, $A = 30^0$, and $g = 9.81$ m/s^2.

43. Use the `switch` structure to write a MATLAB program to compute the amount of money that accumulates in a savings account in one year. The program should accept the following input: the initial amount of money deposited in the account; the frequency of interest compounding (monthly, quarterly, semiannually, or annually); and the interest rate. Run your program for a \$1000 initial deposit for each case; use a 5 percent interest rate. Compare the amounts of money that accumulate for each case.

44. Engineers often need to estimate the pressures and volumes of a gas in a container. The *van der Waals* equation is often used for this purpose. It is

$$P = \frac{RT}{\hat{V} - b} - \frac{a}{\hat{V}^2}$$

where the term b is a correction for the volume of the molecules and the term $a/\hat{V}^2$ is a correction for molecular attractions. The gas constant is R, the *absolute* temperature is T, and the gas specific volume is $\hat{V}$. The value of R is the same for all gases; it is $R = 0.08206$ L-atm/mol-K. The values of a and b depend on the type of gas. Some values are given in the following table. Write a user-defined function using the `switch` structure that computes the pressure P on the basis of the van der Waals equation. The function's input arguments should be T, $\hat{V}$, and a string variable containing the name of a gas listed in the table. Test your function for chlorine (Cl_2) for $T = 300$ K and $\hat{V} = 20$ L/mol.

Gas	a (L^2-atm/mol^2)	b (L/mol)
Helium, He	0.0341	0.0237
Hydrogen, H_2	0.244	0.0266
Oxygen, O_2	1.36	0.0318
Chlorine, Cl_2	6.49	0.0562
Carbon dioxide, CO_2	3.59	0.0427

45. Using the program developed in Problem 19, write a program that uses the `switch` structure to compute the number of days in a year up to a given date, given the year, the month, and the day of the month.