ME 1020 Engineering Programming with MATLAB

Handout 05

Homework 5 Assignment: 5.4, 5.7, 5.10, 5.14, 5.17, 5.20, 5.27, 5.31, 5.34

3.* *a.* Estimate the roots of the equation

$$x^3 - 3x^2 + 5x \sin\left(\frac{\pi x}{4} - \frac{5\pi}{4}\right) + 3 = 0$$

by plotting the equation.

b. Use the estimates found in part *a* to find the roots more accurately with the `fzero` function.

4. To compute the forces in structures, sometimes we must solve equations (called transcendental equations because they have no analytical solution) similar to the following. Plot the function between $0 \le x \le 5$ to roughly locate the zeros of this equation:

$$x \tan x = 9$$

Then use the **fzero** function to accurately find the first three roots. Finally, use the **fplot** function to plot the function in the vicinity of each zero.

5.* Cables are used to suspend bridge decks and other structures. If a heavy uniform cable hangs suspended from its two endpoints, it takes the shape of a *catenary* curve whose equation is

$$y = a \cosh\left(\frac{x}{a}\right)$$

where a is the height of the lowest point on the chain above some horizontal reference line, x is the horizontal coordinate measured to the right from the lowest point, and y is the vertical coordinate measured up from the reference line.

Let $a = 10$ m. Plot the catenary curve for $-20 \leq x \leq 30$ m. How high is each endpoint?

6. Using estimates of rainfall, evaporation, and water consumption, the town engineer developed the following model of the water volume in the reservoir as a function of time

$$V(t) = 10^9 + 10^8(1 - e^{-t/100}) - 10^7 t$$

where V is the water volume in liters and t is time in days. Plot $V(t)$ versus t. Use the plot to estimate how many days it will take before the water volume in the reservoir is 50 percent of its initial volume of 10^9 L.

7. It is known that the following Leibniz series converges to the value $\pi/4$ as $n \to \infty$.

$$S(n) = \sum_{k=0}^{n} (-1)^k \frac{1}{2k + 1}$$

Plot the difference between $\pi/4$ and the sum $S(n)$ versus n for $0 \leq n \leq 200$.

8. A certain fishing vessel is initially located in a horizontal plane at $x = 0$ and $y = 10$ mi. It moves on a path for 10 hr such that $x = t$ and $y = 0.5t^2 + 10$, where t is in hours. An international fishing boundary is described by the line $y = 2x + 6$.

a. Plot and label the path of the vessel and the boundary.

b. The perpendicular distance of the point (x_1, y_1) from the line $Ax + By + C = 0$ is given by

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

where the sign is chosen to make $d \geq 0$. Use this result to plot the distance of the fishing vessel from the fishing boundary as a function of time for $0 \leq t \leq 10$ hr.

9. Plot columns 2 and 3 of the following matrix **A** versus column 1. The data in column 1 are time (seconds). The data in columns 2 and 3 are force (newtons).

$$\mathbf{A} = \begin{bmatrix} 0 & -7 & 6 \\ 5 & -4 & 3 \\ 10 & -1 & 9 \\ 15 & 1 & 0 \\ 20 & 2 & -1 \end{bmatrix}$$

10. Many applications use the following "small angle" approximation for the sine to obtain a simpler model that is easy to understand and analyze. This approximate states that $\sin x \approx x$, where x must be in radians. Investigate the accuracy of this approximation by creating three plots. For the first, plot $\sin x$ versus x for $0 \le x \le 1$. For the second plot the approximation error $\sin x - x$ versus x for $0 \le x \le 1$. For the third plot the relative error $[\sin(x) - x]/\sin(x)$ versus x for $0 \le x \le 1$. How small must x be for the approximation to be accurate within 5 percent? Use the relative error for this calculation.

11. You can use trigonometric identities to simplify the equations that appear in many applications. Confirm the identity $\tan(2x) = 2\tan x/(1 - \tan^2 x)$ by plotting both the left and the right sides versus x over the range $0 \le x \le 2\pi$.

12. The complex number identity $e^{ix} = \cos x + i \sin x$ is often used to convert the solutions of equations into a form that is relatively easy to visualize. Confirm this identity by plotting the imaginary part versus the real part for both the left and right sides over the range $0 \le x \le 2\pi$.

13. Use a plot over the range $0 \le x \le 5$ to confirm that $\sin(ix) = i \sinh x$.

14. The function $y(t) = 1 - e^{-bt}$, where t is time and $b > 0$, describes many processes, such as the height of liquid in a tank as it is being filled and the temperature of an object being heated. Investigate the effect of the parameter b on $y(t)$. To do this, plot y versus t for $0 \le t \le 10$ seconds and $b = 0.01, 0.05, 0.1, 0.5, 1.0, 5.0$ and 10.0 on the same plot. How long will it take for $y(t)$ to reach 98 percent of its steady-state value when $b = 0.5$?

15. The following functions describe the oscillations in electric circuits and the vibrations of machines and structures. Plot these functions on the same plot. Because they are similar, decide how best to plot and label them to avoid confusion.

$$x(t) = 10e^{-0.5t}\sin(3t + 2)$$

$$y(t) = 7e^{-0.4t}\cos(5t - 3)$$

16. In certain kinds of structural vibrations, a periodic force acting on the structure will cause the vibration amplitude to repeatedly increase and decrease with time. This phenomenon, called *beating*, also occurs in musical sounds. A particular structure's displacement is described by

$$y(t) = \frac{1}{f_1^2 - f_2^2}[\cos(f_2 t) - \cos(f_1 t)]$$

where y is the displacement in inches and t is the time in seconds. Plot y versus t over the range $0 \le t \le 20$ for $f_1 = 8$ rad/sec and $f_2 = 1$ rad/sec. Be sure to choose enough points to obtain an accurate plot.

17.* The height $h(t)$ and horizontal distance $x(t)$ traveled by a ball thrown at an angle A with a speed v are given by

$$h(t) = vt \sin A - \frac{1}{2}gt^2$$

$$x(t) = vt \cos A$$

At Earth's surface the acceleration due to gravity is $g = 9.81$ m/s^2.

a. Suppose the ball is thrown with a velocity $v = 10$ m/s at an angle of 35°. Use MATLAB to compute how high the ball will go, how far it will go, and how long it will take to hit the ground.

b. Use the values of v and A given in part *a* to plot the ball's *trajectory;* that is, plot h versus x for positive values of h.

c. Plot the trajectories for $v = 10$ m/s corresponding to five values of the angle A: 20°, 30°, 45°, 60°, and 70°.

d. Plot the trajectories for $A = 45°$ corresponding to five values of the initial velocity v: 10, 12, 14, 16, and 18 m/s.

18. The perfect gas law relates the pressure p, absolute temperature T, mass m, and volume V of a gas. It states that

$$pV = mRT$$

The constant R is the *gas constant*. The value of R for air is 286.7 (N · m)/(kg · K). Suppose air is contained in a chamber at room temperature (20°C = 293 K). Create a plot having three curves of the gas pressure in N/m^2 versus the container volume V in m^3 for $20 \le V \le 100$. The three curves correspond to the following masses of air in the container: $m = 1$ kg, $m = 3$ kg, and $m = 7$ kg.

19. Oscillations in mechanical structures and electric circuits can often be described by the function

$$y(t) = e^{-t/\tau} \sin(\omega t + \phi)$$

where t is time and ω is the oscillation frequency in radians per unit time. The oscillations have a period of $2\pi/\omega$, and their amplitudes decay in time at a rate determined by τ, which is called the *time constant*. The smaller τ is, the faster the oscillations die out.

a. Use these facts to develop a criterion for choosing the spacing of the t values and the upper limit on t to obtain an accurate plot of $y(t)$. (*Hint:* Consider two cases: $4\tau > 2\pi/\omega$ and $4\tau < 2\pi/\omega$.)

b. Apply your criterion, and plot $y(t)$ for $\tau = 10$, $\omega = \pi$, and $\phi = 2$.

c. Apply your criterion, and plot $y(t)$ for $\tau = 0.1$, $\omega = 8\pi$, and $\phi = 2$.

20. When a constant voltage was applied to a certain motor initially at rest, its rotational speed $s(t)$ versus time was measured. The data appear in the following table:

Time (sec)	1	2	3	4	5	6	7	8	10
Speed (rpm)	1210	1866	2301	2564	2724	2881	2879	2915	3010

Plot the rotational speed versus time using open red circles. Then plot the following function on the same graph:

$$s(t) = b(1 - e^{-ct}); \qquad b = 3010, \qquad c = 0.1{:}\,0.1{:}\,0.6$$

What value of c provides the best fit to the data?

21. The following table shows the average temperature for each year in a certain city. Plot the data as a stem plot, a bar plot, and a stairs plot.

Year	2000	2001	2002	2003	2004
Temperature (°C)	21	18	19	20	17

22. A sum of \$10 000 invested at 4 percent interest compounded annually will grow according to the formula

$$y(k) = 10^4(1.04)^k$$

where k is the number of years ($k = 0, 1, 2, \ldots$). Plot the amount of money in the account for a 10-year period. Do this problem with four types of plots: the *xy* plot, the stem plot, the stairs plot, and the bar plot.

23. The volume V and surface area A of a sphere of radius r are given by

$$V = \frac{4}{3}\pi r^3 \qquad A = 4\pi r^2$$

a. Plot V and A versus r in two subplots, for $0.1 \leq r \leq 100$ m. Choose axes that will result in straight-line graphs for both V and A.
b. Plot V and r versus A in two subplots, for $1 \leq A \leq 10^4$ m^2. Choose axes that will result in straight-line graphs for both V and r.

24. The current amount A of a principal P invested in a savings account paying an annual interest rate r is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where n is the number of times per year the interest is compounded. For continuous compounding, $A = Pe^{rt}$. Suppose \$10 000 is initially invested at 3.5 percent ($r = 0.035$).
a. Plot A versus t for $0 \leq t \leq 20$ years for four cases: continuous compounding, annual compounding ($n = 1$), quarterly compounding ($n = 4$), and monthly compounding ($n = 12$). Show all four cases on the same subplot and label each curve. On a second subplot, plot the

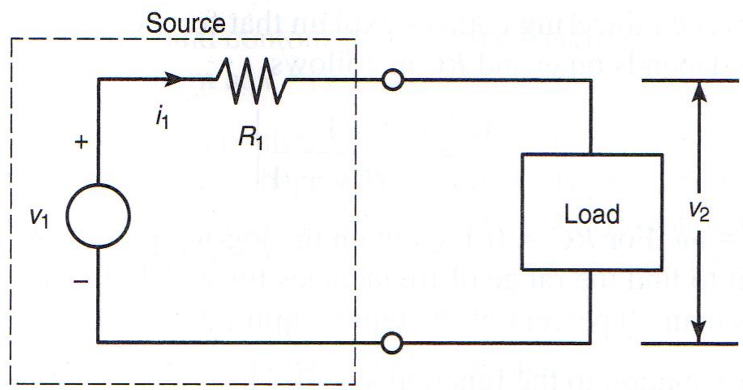

Figure P25

difference between the amount obtained from continuous compounding and the other three cases.

b. Redo part *a*, but plot *A* versus *t* on log-log and semilog plots. Which plot gives a straight line?

25. Figure P25 is a representation of an electrical system with a power supply and a load. The power supply produces the fixed voltage v_1 and supplies the current i_1 required by the load, whose voltage drop is v_2. The current–voltage relationship for a specific load is found from experiments to be

$$i_1 = 0.16(e^{0.12v_2} - 1)$$

Suppose that the supply resistance is $R_1 = 30\ \Omega$ and the supply voltage is $v_1 = 15$ V. To select or design an adequate power supply, we need to determine how much current will be drawn from the power supply when this load is attached. Find the voltage drop v_2 as well.

26. The circuit shown in Figure P26 consists of a resistor and a capacitor and is thus called an *RC* circuit. If we apply a sinusoidal voltage v_i, called the input voltage, to the circuit as shown, then eventually the output voltage v_o will be sinusoidal also, with the same frequency but with a different amplitude and shifted in time relative to the input voltage. Specifically, if $v_i = A_i \sin \omega t$, then $v_o = A_o \sin(\omega t + \phi)$. The frequency–response plot is a plot of A_o/A_i versus frequency ω. It is usually plotted on logarithmic axes.

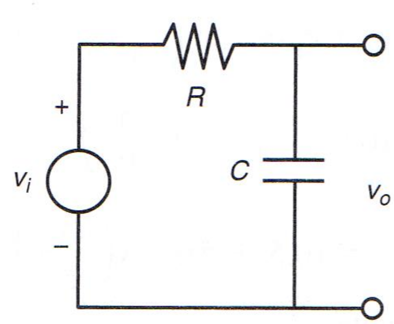

Figure P26

Upper-level engineering courses explain that for the RC circuit shown, this ratio depends on ω and RC as follows:

$$\frac{A_o}{A_i} = \left| \frac{1}{RCs + 1} \right|$$

where $s = \omega i$. For $RC = 0.1$ s, obtain the log-log plot of $|A_o/A_i|$ versus ω and use it to find the range of frequencies for which the output amplitude A_o is less than 70 percent of the input amplitude A_i.

27. An approximation to the function $\sin x$ is $\sin x \approx x - x^3/6$. Plot the $\sin x$ function and 20 evenly spaced error bars representing the error in the approximation.

Section 5.4

28. The popular amusement ride known as the corkscrew has a helical shape. The parametric equations for a circular helix are

$$x = a \cos t$$
$$y = a \sin t$$
$$z = bt$$

where a is the radius of the helical path and b is a constant that determines the "tightness" of the path. In addition, if $b > 0$, the helix has the shape of a right-handed screw; if $b < 0$, the helix is left-handed.

Obtain the three-dimensional plot of the helix for the following three cases and compare their appearance with one another. Use $0 \le t \le 10\pi$ and $a = 1$.

a. $b = 0.1$
b. $b = 0.2$
c. $b = -0.1$

29. A robot rotates about its base at 2 rpm while lowering its arm and extending its hand. It lowers its arm at the rate of 120° per minute and extends its hand at the rate of 5 m/min. The arm is 0.5 m long. The xyz coordinates of the hand are given by

$$x = (0.5 + 5t) \sin\left(\frac{2\pi}{3}t\right) \cos(4\pi t)$$

$$y = (0.5 + 5t) \sin\left(\frac{2\pi}{3}t\right) \sin(4\pi t)$$

$$z = (0.5 + 5t) \cos\left(\frac{2\pi}{3}t\right)$$

where t is time in minutes.

Obtain the three-dimensional plot of the path of the hand for $0 \le t \le 0.2$ min.

30. Obtain the surface and contour plots for the function $z = x^2 - 2xy + 4y^2$, showing the minimum at $x = y = 0$.

31. Obtain the surface and contour plots for the function $z = -x^2 + 2xy + 3y^2$. This surface has the shape of a saddle. At its saddlepoint at $x = y = 0$, the surface has zero slope, but this point does not correspond to either a minimum or a maximum. What type of contour lines corresponds to a saddlepoint?

32. Obtain the surface and contour plots for the function $z = (x - y^2)(x - 3y^2)$. This surface has a singular point at $x = y = 0$, where the surface has zero slope, but this point does not correspond to either a minimum or a maximum. What type of contour lines corresponds to a singular point?

33. A square metal plate is heated to 80°C at the corner corresponding to $x = y = 1$. The temperature distribution in the plate is described by

$$T = 80e^{-(x-1)^2}e^{-3(y-1)^2}$$

Obtain the surface and contour plots for the temperature. Label each axis. What is the temperature at the corner corresponding to $x = y = 0$?

34. The following function describes oscillations in some mechanical structures and electric circuits.

$$z(t) = e^{-t/\tau}\sin(\omega t + \phi)$$

In this function t is time, and ω is the oscillation frequency in radians per unit time. The oscillations have a period of $2\pi/\omega$, and their amplitudes decay in time at a rate determined by τ, which is called the *time constant.* The smaller τ is, the faster the oscillations die out.

Suppose that $\phi = 0$, $\omega = 2$, and τ can have values in the range $0.5 \le \tau \le 10$ sec. Then the preceding equation becomes

$$z(t) = e^{-t/\tau}\sin(2t)$$

Obtain a surface plot and a contour plot of this function to help visualize the effect of τ for $0 \le t \le 15$ sec. Let the x variable be time t and the y variable be τ.

35. The following equation describes the temperature distribution in a flat rectangular metal plate. The temperature on three sides is held constant at T_1, and at T_2 on the fourth side (see Figure P35). The temperature $T(x, y)$ as a function of the xy coordinates shown is given by

$$T(x,y) = (T_2 - T_1)w(x,y) + T_1$$

where

$$w(x,y) = \frac{2}{\pi}\sum_{n\ \mathrm{odd}}^{\infty}\frac{2}{n}\sin\left(\frac{n\pi x}{L}\right)\frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$$

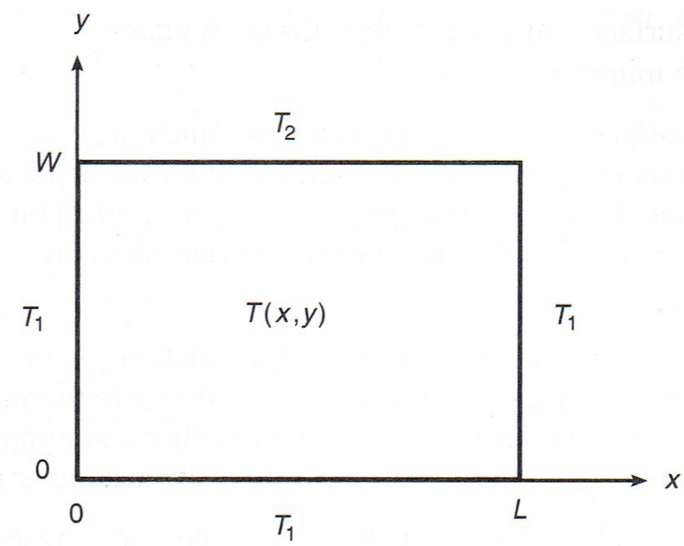

Figure P35

The given data for this problem are $T_1 = 70°F$, $T_2 = 200°F$, and $W = L = 2$ ft.

Using a spacing of 0.2 for both x and y, generate a surface mesh plot and a contour plot of the temperature distribution.

36. The electric potential field V at a point, due to two charged particles, is given by

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

where q_1 and q_2 are the charges of the particles in coulombs (C), r_1 and r_2 are the distances of the charges from the point (in meters), and ϵ_0 is the permittivity of free space, whose value is

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$

Suppose the charges are $q_1 = 2 \times 10^{-10}$ C and $q_2 = 4 \times 10^{-10}$ C. Their respective locations in the xy plane are (0.3, 0) and (−0.3, 0) m. Plot the electric potential field on a three-dimensional surface plot with V plotted on the z axis over the ranges $-0.25 \le x \le 0.25$ and $-0.25 \le y \le 0.25$. Create the plot in two ways: (*a*) by using the `surf` function and (*b*) by using the `meshc` function.

37. Refer to Problem 25 of Chapter 4. Use the function file created for that problem to generate a surface mesh plot and a contour plot of x versus h and W for $0 \le W \le 500$ N and for $0 \le h \le 2$ m. Use the values $k_1 = 10^4$ N/m, $k_2 = 1.5 \times 10^4$ N/m, and $d = 0.1$ m.

38. Refer to Problem 28 of Chapter 4. To see how sensitive the cost is to location of the distribution center, obtain a surface plot and a contour plot of the total cost as a function of the x and y coordinates of the distribution center location. How much would the cost increase if we located the center 1 mi in any direction from the optimal location?

39. Refer to Example 3.2–1. Use a surface plot and a contour plot of the perimeter length L as a function of d and θ over the ranges $1 \leq d \leq 30$ ft and $0.1 \leq \theta \leq 1.5$ rad. Are there valleys other than the one corresponding to $d = 7.5984$ and $\theta = 1.0472$? Are there any saddle points?