ME 1020 Engineering Programming with MATLAB

Handout 06

Homework 6 Assignment: 6.2, 6.7, 6.12, 6.16

Section 6.1

1. The distance a spring stretches from its "free length" is a function of how much tension force is applied to it. The following table gives the spring length y that the given applied force f produced in a particular spring. The spring's free length is 4.7 in. Find a functional relation between f and x, the extension from the free length ($x = y - 4.7$).

Force f (lb)	Spring length y (in.)
0	4.7
0.94	7.2
2.30	10.6
3.28	12.9

2.* In each of the following problems, determine the best function $y(x)$ (linear, exponential, or power function) to describe the data. Plot the function on the same plot with the data. Label and format the plots appropriately.

a.

x	25	30	35	40	45
y	5	260	480	745	1100

b.

x	2.5	3	3.5	4	4.5	5	5.5	6	7	8	9	10
y	1500	1220	1050	915	810	745	690	620	520	480	410	390

c.

x	550	600	650	700	750
y	41.2	18.62	8.62	3.92	1.86

3. The population data for a certain country are as follows:

Year	2004	2005	2006	2007	2008	2009
Population (millions)	10	10.9	11.7	12.6	13.8	14.9

Obtain a function that describes these data. Plot the function and the data on the same plot. Estimate when the population will be double its 2004 size.

4.* The *half-life* of a radioactive substance is the time it takes to decay by one-half. The half-life of carbon 14, which is used for dating previously living things, is 5500 years. When an organism dies, it stops accumulating carbon 14. The carbon 14 present at the time of death decays with time. Let $C(t)/C(0)$ be the fraction of carbon 14 remaining at time t. In radioactive carbon dating, scientists usually assume that the remaining fraction decays exponentially according to the following formula:

$$\frac{C(t)}{C(0)} = e^{-bt}$$

a. Use the half-life of carbon 14 to find the value of the parameter b, and plot the function.
b. If 90 percent of the original carbon 14 remains, estimate how long ago the organism died.
c. Suppose our estimate of b is off by ± 1 percent. How does this error affect the age estimate?

5. *Quenching* is the process of immersing a hot metal object in a bath for a specified time to obtain certain properties such as hardness. A copper sphere 25 mm in diameter, initially at 300°C, is immersed in a bath at 0°C. The following table gives measurements of the sphere's temperature versus time. Find a functional description of these data. Plot the function and the data on the same plot.

Time (s)	0	1	2	3	4	5	6
Temperature (°C)	300	150	75	35	12	5	2

6. The useful life of a machine bearing depends on its operating temperature, as the following data show. Obtain a functional description of these data. Plot the function and the data on the same plot. Estimate a bearing's life if it operates at 150°F.

Temperature (°F)	100	120	140	160	180	200	220
Bearing life (hours × 10^3)	28	21	15	11	8	6	4

7. A certain electric circuit has a resistor and a capacitor. The capacitor is initially charged to 100 V. When the power supply is detached, the capacitor voltage decays with time, as the following data table shows. Find a functional description of the capacitor voltage v as a function of time t. Plot the function and the data on the same plot.

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5
Voltage (V)	100	62	38	21	13	7	4	2

Sections 6.2 and 6.3

8.* The distance a spring stretches from its free length is a function of how much tension force is applied to it. The following table gives the spring length y that was produced in a particular spring by the given applied force f. The spring's free length is 4.7 in. Find a functional relation between f and x, the extension from the free length ($x = y - 4.7$).

Force f (lb)	Spring length y (in.)
0	4.7
0.94	7.2
2.30	10.6
3.28	12.9

9. The following data give the drying time T of a certain paint as a function of the amount of a certain additive A.

a. Find the first-, second-, third-, and fourth-degree polynomials that fit the data, and plot each polynomial with the data. Determine the quality of the curve fit for each by computing J, S, and r^2.

b. Use the polynomial giving the best fit to estimate the amount of additive that minimizes the drying time.

A (oz)	0	1	2	3	4	5	6	7	8	9
T (min)	130	115	110	90	89	89	95	100	110	125

10. The following data give the stopping distance d as a function of the initial speed v, for a certain car model. Using the **polyfit** command, find a third-order polynomial that fits the data. Show the original data and the curve fit on a plot. Using the curve fit, estimate the stopping distance for an initial speed of 63 mi/hr.

v (mi/hr)	20	30	40	50	60	70
d (ft)	45	80	130	185	250	330

11.* The number of twists y required to break a certain rod is a function of the percentage x_1 and x_2 of each of two alloying elements present in the rod. The following table gives some pertinent data. Use linear multiple regression to obtain a model $y = a_0 + a_1x_1 + a_2x_2$ of the relationship between the number of twists and the alloy percentages. In addition, find the maximum percent error in the predictions.

Number of twists y	Percentage of element 1 x_1	Percentage of element 2 x_2
40	1	1
51	2	1
65	3	1
72	4	1
38	1	2
46	2	2
53	3	2
67	4	2
31	1	3
39	2	3
48	3	3
56	4	3

12. The following represents pressure samples, in pounds per square inch (psi), taken in a fuel line once every second for 10 seconds. Fit a first-degree polynomial, a second-degree polynomial, and a third-degree polynomial to these data using the **polyfit** command. Plot the curve fits along with the original data. Use the third-degree polynomial curve fit to provide an estimate of the pressure at $t = 11$ seconds.

Time (sec)	Pressure (psi)	Time (sec)	Pressure (psi)
1	26.1	6	30.6
2	27.0	7	31.1
3	28.2	8	31.3
4	29.0	9	31.0
5	29.8	10	30.5

13. Data on the vapor pressure P of water as a function of temperature T are given in the following table. From theory we know that $\ln P$ is proportional to $1/T$. Obtain a curve fit for $P(T)$ from these data using the **Basic Fitting Interface**. Use the fit to estimate the vapor pressure at $T = 285$ K.

T (K)	P (torr)
273	4.579
278	6.543
283	9.209
288	12.788
293	17.535
298	23.756

14. The solubility of salt in water is a function of the water temperature. Let S represent the solubility of NaCl (sodium chloride) as grams of salt in 100 g of water. Let T be temperature in °C. Use the following data to obtain a curve fit for S as a function of T. Use the fit to estimate S when $T = 25$°C.

T (°C)	S (g NaCl/100 g H_2O)
10	35
20	35.6
30	36.25
40	36.9
50	37.5
60	38.1
70	38.8
80	39.4
90	40

15. The solubility of oxygen in water is a function of the water temperature. Let S represent the solubility of O_2 as millimoles of O_2 per liter of water. Let T be temperature in °C. Use the following data to obtain a curve fit for S as a function of T. Use the fit to estimate S when $T = 8$°C and $T = 50$°C.

T (°C)	S (mmol O_2 /L H_2O)
5	1.95
10	1.7
15	1.55
20	1.40
25	1.30
30	1.15
35	1.05
40	1.00
45	0.95

16. The following function is linear in the parameters a_1 and a_2.

$$y(x) = a_1 + a_2 \ln x$$

Use the **polyfit** command with the following data to estimate the values of a_1 and a_2. Use the curve fit to estimate the values of y at x = 2.5 and at x = 11. Use the **Basic Fitting Interface** to determine a fourth-order polynomial fit to the data and estimate the values of y at x = 2.5 and at x = 11.

x	1	2	3	4	5	6	7	8	9	10
y	10	14	16	18	19	20	21	22	23	23

17. Chemists and engineers must be able to predict the changes in chemical concentration in a reaction. A model used for many single-reactant processes is

$$\text{Rate of change of concentration} = -kC^n$$

where C is the chemical concentration and k is the rate constant. The order of the reaction is the value of the exponent n. Solution methods for differential equations (which are discussed in Chapter 9) can show that the solution for a first-order reaction ($n = 1$) is

$$C(t) = C(0)e^{-kt}$$

The following data describe the reaction

$$(CH_3)_3CBr + H_2O \rightarrow (CH_3)_3COH + HBr$$

Use these data to obtain a least-squares fit to estimate the value of k.

Time t (h)	C (mol of $(CH3)_3$ CBr/L)
0	0.1039
3.15	0.0896
6.20	0.0776
10.0	0.0639
18.3	0.0353
30.8	0.0207
43.8	0.0101

18. Chemists and engineers must be able to predict the changes in chemical concentration in a reaction. A model used for many single-reactant processes is

$$\text{Rate of change of concentration} = -kC^n$$

where C is the chemical concentration and k is the rate constant. The order of the reaction is the value of the exponent n. Solution methods for differential equations (which are discussed in Chapter 9) can show that the solution for a first-order reaction ($n = 1$) is

$$C(t) = C(0)e^{-kt}$$

and the solution for a second-order reaction ($n = 2$) is

$$\frac{1}{C(t)} = \frac{1}{C(0)} + kt$$

The following data (from Brown, 1994) describes the gas-phase decomposition of nitrogen dioxide at 300°C.

$$2NO_2 \rightarrow 2NO + O_2$$

Time t (s)	C (mol NO_2 /L)
0	0.0100
50	0.0079
100	0.0065
200	0.0048
300	0.0038

Determine whether this is a first-order or second-order reaction, and estimate the value of the rate constant k.

19. Chemists and engineers must be able to predict the changes in chemical concentration in a reaction. A model used for many single-reactant processes is

$$\text{Rate of change of concentration} = -kC^n$$

where C is the chemical concentration and k is the rate constant. The order of the reaction is the value of the exponent n. Solution methods for differential equations (which are discussed in Chapter 9) can show that the solution for a first-order reaction ($n = 1$) is

$$C(t) = C(0)e^{-kt}$$

The solution for a second-order reaction ($n = 2$) is

$$\frac{1}{C(t)} = \frac{1}{C(0)} + kt$$

and the solution for a third-order reaction ($n = 3$) is

$$\frac{1}{2C^2(t)} = \frac{1}{2C^2(0)} + kt$$

Time t (min)	C (mol of reactant/L)
5	0.3575
10	0.3010
15	0.2505
20	0.2095
25	0.1800
30	0.1500
35	0.1245
40	0.1070
45	0.0865

The preceding data describe a certain reaction. By examining the residuals, determine whether this is a first-order, second-order, or third-order reaction, and estimate the value of the rate constant k.